



Solute-Transport under Fluctuating Groundwater Flow in Homogeneous Finite Porous Domain

Dilip Kumar Jaiswal¹, R. R. Yadav^{1*} and Gulrana¹

Abstract

In this paper a theoretical model is developed for the advection-dispersion problem in one-dimensional porous media with two considerations: one the flow is periodic and the second dispersion coefficient is directly proportional to the seepage velocity. The porous domain is homogeneous, isotropic and of adsorbing nature. A time dependent periodic point source is considered at the source boundary. Different boundary conditions are considered at outlet of the domain. In first case, the mixed type and in second case flux type boundary conditions are considered. For both cases, input source are same. We studied the influence on concentration profiles due to different boundary conditions in the domain. The derived solution is also extended in semi-infinite domain. The Laplace Transformation Technique (LTT) is used to get analytical solution. In this process, a new time variables are introduced. Graphical illustrations of concentration profiles versus time and position are presented for different set of data.

Keywords

Groundwater; Pollution; Analytical solution; Hydrodynamic dispersion

Introduction

In recent years, problems which involve the flow of water and solutes separately or simultaneously through saturated/unsaturated porous media have received much attention in different field of science and technology. Advection-dispersion equation is applicable in many disciplines like groundwater hydrology, chemical engineering, bio sciences, environmental sciences and petroleum engineering. The transport of solute in porous media depends on several factors including the solute properties, the seepage flow velocity within the porous medium and shape, size and location of solid and voids part of the medium. Subsurface water flow is normally independent of the contaminant being transported. Some solutes are non-reactive and therefore act as passive tracers, moving with the water as it advects and disperses in the subsurface. The flow of solute and the flow of water through the porous media is considered as a bulk transport problem which can be described through the use of differential equation known as advection-dispersion equation.

*Corresponding author: R. R. Yadav, PhD, Department of Mathematics and Astronomy, Lucknow University, Lucknow-226007, India, E-mail: yadav_rr2@yahoo.co.in

Received: October 18, 2012 Accepted: January 02, 2013 Published: January 07, 2013

The literature on solute transport in porous media is voluminous. The majority of the available analytical solutions of advection-dispersion are derived by taking into account seepage velocity steady, unsteady and boundary condition non-periodic. Ferris [1] derived a solution for the steady periodic response of the piezometric heads in a semi-infinite aquifer subject to a sinusoidal variation of the stream water level. Field studies of the groundwater recharge/discharge process in aquifers (especially in coastal area) show that the tide can importantly regulate the temporal and spatial blueprint of groundwater recharge/discharge and the salt concentration in the near-shore groundwater. Kumar [2] considered the flow velocity unsteady/non-uniform in homogeneous porous domain.

Rao and Sharma [3] derived an analytical solution using Fourier transforms to determine a groundwater profile under the influence of local recharge to an unconfined aquifer. Kinzelbach and Ackerer [4] studied that in steady flow, condition temporal variations yields a larger transverse dispersivity, in comparison to longitudinal dispersivity. A temporal fluctuation in the recharge or in the boundary conditions causes variations in the groundwater velocity field that lead to dispersive mixing. Gelhar [5] studied the effect of spatial variability on the solutes spreading in porous media. Goode and Konikow demonstrated that directional variation in hydraulic gradient is more important than the magnitude variation in the hydraulic gradient [6]. Webster and Taylor observed that groundwater flow direction rotates with time due to variation in pressure gradient [7].

Several other analytical solutions have been derived to describe the periodic groundwater flow demonstrated that unsteady seepage velocity which influences the rainfall infiltration and water level variation with seepage flow is based on two-phase flow concept [8-10]. Point source contaminant generally supposed to leach in groundwater table continuously. However, seasonal variations in the recharge and level of groundwater table, temporal/spatial variation in source concentration, heterogeneous nature of source etc. may highly cause to leach contaminant continuously. Jaiswal et al., Kumar et al. and Yadav et al. [11-14] obtained analytical solutions for advection-diffusion equation with variable coefficients, for temporally and spatially dependent dispersion problems. Pérez et al. [15] presented an exact solution of advection-diffusion equation with constant coefficients for both transient and steady-state regimes and solved analytically using the Generalized Integral Transform Technique (GITT). Chen and Chen and Liu derived analytical solutions for one-dimensional advective-dispersive transport in finite spatial domain with three simple time-dependent inlet conditions including constant, exponentially decaying and sinusoidally periodic input functions and demonstrate the applicability of solution [16].

The cause of the variations in flow velocity is normally assigned to variations in hydraulic conductivity. To correctly represent a dispersion model several key factors should include cause variations in velocity and those caused by a fluctuating boundary condition. The purpose of this study is to derive analytical solutions to one-dimensional advection-dispersion equation with periodic flow along two set of boundary conditions resulting from time dependent periodic injection. In present study, we assume the aquifer is homogeneous,

isotropic and of adsorbing nature. Initially the domain is not solute free. Solutions are derived for finite domain and later extended for semi-infinite domain. The practical usefulness of the effects of various physical parameters and their importance is also discussed.

Mathematical Formulation of the Problem

The mathematical formulation and analysis starts by investigating one-dimensional groundwater flow through porous media. As generally known, the mass transport equation uses hydrodynamic dispersion, which is the combination of mechanical dispersion and diffusion. The advection-dispersion equation in one-dimension can be written as [17,18]:

$$\frac{\partial c}{\partial t} + \frac{1-n_p}{n_p} \frac{\partial F}{\partial t} = \frac{\partial}{\partial x} \left(D(x,t) \frac{\partial c}{\partial x} - u(x,t)c \right) \quad (1)$$

In which c is the solute concentration in the liquid phase [$M L^{-2}$] and F is the concentration in the solid phase [$M L^{-2}$]. The dispersion coefficient, D presumably includes the effects of both molecular diffusion and mixing in the axial direction; however molecular diffusion is negligible due to very low seepage velocity. The first and second terms on the left side of the equation (1) represent change in concentration with time in liquid and solid phase respectively where n_p denotes porosity and first term of the right-hand side of the equation (1) describes the influence of the dispersion on the concentration distribution. The second term shows the change of the concentration due to advective transport. In equation (1), D and u may be constants or functions of time or space. Velocity fluctuations at the scale of pores cause a solute particle to spread from initial position. This spreading phenomenon is described at the Darcy scale through dispersion coefficients.

The model simulates concentration along one-dimensional periodic flow through homogeneous finite porous medium. Periodic groundwater flow is considered along the x -axis which means the direction of the flow of water is from $x=0$ to $x=L$. Initially the porous domain is not supposed to be solute free, it means before solute injection in the domain there are some concentrations already present in the domain. A periodic mass injection of solute is introduced into the aquifer at $x=0$ over the interval $0 \leq x \leq L$. Groundwater advection carries the mass of solute with it. In the process, the solute slug spreads out so that the maximum concentration decreases with time. The groundwater flow in the aquifer is transient where the velocity follows periodically. The periodic form of velocity may represent the seasonal variation in a year in tropical regions. The influence of water level fluctuations on the solute transport under proposed conditions is investigated analytically.

In 1952, Lapidus and Amundson [19] considered two cases, namely,

$$F = K_1 c + K_2 \quad (2a)$$

and

$$\frac{\partial F}{\partial t} = K_1 c - K_2 F \quad (2b)$$

for equilibrium and non-equilibrium relationship between the concentrations in two phases respectively, where K_1 and K_2 are empirical constants. For simplicity, the former relationship is adopted in the present analysis. The relation (2b) makes the differential equation non-linear, so the analytical solution will not be possible.

Equation (1) can be written as

$$\text{or } R \frac{\partial c}{\partial t} = \frac{\partial}{\partial x} \left(D(x,t) \frac{\partial c}{\partial x} - u(x,t)c \right) \quad (3)$$

where $R = \left(1 + \frac{1-n_p}{n_p} K_1 \right)$ is a retardation factor describing solute sorption. The coefficient of dispersion is considered directly proportional to seepage velocity [20] i.e.,

$$D(x,t) \propto |u(x,t)| \quad (4)$$

Let us write $u(x,t) = u_0 |\sin(mt)|$, and $D(x,t) = D_0 |\sin(mt)|$, where D_0 is initial dispersion coefficient, when $mt=0$, $D(x,t)=0$ But $D(x,t)=D_0$ at $mt=\pi/2$ i.e., $D(x,t)$ represents the peak value of $|D(x,t)|$. Similarly u_0 may be interpreted. So Eq. (1) is valid in $t>0$ domain. m is a unsteady parameter whose dimension is inverse to that the time variable t , thus $\sin(mt)$ is an expression of non-dimensional variable. So equation (3) becomes,

$$R \frac{\partial c}{\partial t} = \frac{\partial}{\partial x} \left(D_0 |\sin(mt)| \frac{\partial c}{\partial x} - u_0 |\sin(mt)| c \right) \quad (5)$$

where D_0 , u_0 are constants and the dimension of coefficients D_0 , u_0 are $L^2 T^{-1}$, $L T^{-1}$ respectively. Initially the domain is not solute free. Let us assume it is an exponential increasing function of space variable. Source concentration of periodic nature is assumed at the origin of the domain which means the pollutant disperse along the flow.

Case-I: Mixed type boundary condition at the extreme end

Under the proposed assumptions, mathematically the initial and boundary conditions in a finite domain for mixed type boundary condition may be written as follows:

$$C(x,t) = C_i \exp(\alpha x), \quad 0 \leq x \leq L, \quad t=0 \quad (6)$$

$$c(x,t) = C_0 \{1 + \cos(mt)\}, \quad x=0, \quad t>0 \quad (7a)$$

$$\frac{\partial c}{\partial x} = \frac{u_0}{2D_0} c, \quad x=L, \quad t>0 \quad (7b)$$

where C_i is the resident concentration [$M L^{-2}$] and α is the parameter taken less than one and its dimension is inverse of space variable.

The practical significance of the boundary condition (7a) is the periodic concentration at source of the boundary i.e., $x=0$. The field observations indicate the source concentration may not be negative, therefore $C_0 \{1 + \cos(mt)\}$ is taken. The significance of condition (7b) is, it allows for contaminant concentration at the influent boundary to be lower than the influent concentration initially.

Let us introduce a new time variable using the following transformation [21],

$$T = \int_0^t |\sin(mt)| dt = \int_0^t \sin(mt) dt \quad \text{as } |\sin(mt)| = \sin(mt) \quad \text{in } t>0 \quad \text{domain.}$$

$$\text{or } T = \frac{1}{m} \{1 - \cos(mt)\} \quad (8)$$

Therefore differential equation (5) with initial and boundary conditions (6-7) can be written as,

$$R \frac{\partial c}{\partial T} = \frac{\partial}{\partial x} \left(D_0 \frac{\partial c}{\partial x} - u_0 c \right) \quad (9)$$

$$c(x,T) = C_i \exp(\alpha x), \quad 0 \leq x \leq L, \quad T=0 \quad (10)$$

$$c(x, T) = C_0(2 - mT), \quad x=0, T > 0 \tag{11a}$$

$$\frac{\partial c}{\partial x} = \frac{u_0}{2D_0} c, \quad x=L, T > 0 \tag{11b}$$

Now introducing a transformation,

$$c(x, T) = K(x, T) \exp\left[\frac{u_0}{2D_0}x - \frac{u_0^2}{4RD_0}T\right] \tag{12}$$

Equations (9)–(11) reduced into

$$R \frac{\partial K}{\partial T} = D_0 \frac{\partial^2 K}{\partial x^2} \tag{13}$$

$$K(x, T) = C_i \exp(\alpha x - \beta x), \quad 0 \leq x \leq L, T=0; \quad \beta = \frac{u_0}{2D_0} \tag{14}$$

$$K(x, T) = C_0(2 - mT) \exp(\gamma^2 T), \quad x=0, T > 0; \quad \gamma^2 = \frac{u_0^2}{4RD_0} \tag{15a}$$

$$\frac{\partial K}{\partial x} = 0, \quad x=L, T > 0 \tag{15b}$$

Applying Laplace transformation on equations. (13–15), we get

$$Rp \bar{K} - C_i R \exp(\alpha x - \beta x) = D_0 \frac{d^2 \bar{K}}{dx^2} \tag{16}$$

$$\bar{K}(x, p) = C_0 \left\{ \frac{2}{(p - \gamma^2)} - \frac{m}{(p - \gamma^2)^2} \right\}, \quad x=0 \tag{17a}$$

$$\frac{d\bar{K}}{dx} = 0, \quad x=L \tag{17b}$$

where $\bar{K}(x, p) = \int_0^\infty K(x, T) e^{-pT} dT$ and p is the Laplace transformation parameter.

Thus the general solution of equation (16) may be written as

$$\bar{K}(x, p) = C_1 \cosh(Mx) + C_2 \sinh(Mx) + \frac{C_i \exp(\alpha - \beta)x}{\left\{ p - \frac{D_0}{R}(\alpha - \beta)^2 \right\}}, \tag{18}$$

$$M = \sqrt{\frac{pR}{D_0}}$$

Using conditions (17a and b) on the above solution, we get

$$C_1 = C_0 \left\{ \frac{2}{(p - \gamma^2)} - \frac{m}{(p - \gamma^2)^2} \right\} - \frac{C_i}{\left\{ p - \frac{D_0}{R}(\alpha - \beta)^2 \right\}}$$

and

$$C_2 = -C_0 \left\{ \frac{2}{(p - \gamma^2)} - \frac{m}{(p - \gamma^2)^2} \right\} \frac{\sinh hML}{\cosh hML} - \frac{C_i}{\left\{ p - \frac{D_0}{R}(\alpha - \beta)^2 \right\}} \left\{ \frac{(\alpha - \beta) \exp\{(\alpha - \beta)L\}}{M \cosh hML} - \frac{\sinh hML}{\cosh hML} \right\}$$

Substituting the values of C_1 and C_2 in equation (18), we may get,

$$\bar{K}(x, p) = \left[C_0 \left\{ \frac{2}{(p - \gamma^2)} - \frac{m}{(p - \gamma^2)^2} \right\} \frac{\cosh(Mx - ML)}{\cosh(LM)} \right.$$

$$\left. - \frac{C_i}{\left\{ p - \frac{D_0}{R}(\alpha - \beta)^2 \right\}} \left\{ \frac{\cosh(Mx - ML)}{\cosh hML} + \frac{(\alpha - \beta) \exp\{(\alpha - \beta)L\} \sinh(Mx)}{M \cosh hML} \right\} \right] + \frac{C_i \exp(\alpha - \beta)x}{\left\{ p - \frac{D_0}{R}(\alpha - \beta)^2 \right\}} \tag{19}$$

Taking inverse Laplace transform of equation (19), the analytical solution of advection-dispersion solute transport for periodic input condition in terms of $c(x, T)$, as

$$c(x, t) = C_0 \exp\{x\beta\} \left[2 \frac{\cosh(x-L)\beta}{\cosh(L)\beta} + \left\{ L \frac{mR}{u_0} \tanh(L)\beta - mT \right\} \right.$$

$$\left. \frac{\cosh(x-L)\beta}{\cosh(L)\beta} - \frac{mR(x-L)}{u_0} \frac{\sinh(x-L)\beta}{\cosh(L)\beta} \right]$$

$$- 2\pi C_0 D_0 \exp\{x\beta - \gamma^2 T\} \left[\sum_0^\infty \left\{ 2 + \frac{mR(L)^2}{\left\{ (n + 1/2)^2 \pi^2 D_0 + \gamma^2 R(L)^2 \right\}} \right\} \frac{F_1}{F_2} E \right]$$

$$- C_i \exp(\beta x - \gamma^2 T) \left\{ e^{bT} \frac{\cosh(x-L) \sqrt{bR/D_0}}{\cosh(L) \sqrt{bR/D_0}} - 2\pi D_0 \sum_{n=0}^\infty \frac{F_1}{F_3} E \right\}$$

$$- C_i (\alpha - \beta) \exp(\alpha L - \beta L + \beta x - \gamma^2 T) \left[\frac{e^{bT}}{\sqrt{bR/D_0}} \frac{\sinh(x) \sqrt{bR/D_0}}{\cosh(L) \sqrt{bR/D_0}} \right.$$

$$\left. - 2LD_0 \sum_{n=0}^\infty (-1)^n \sin \left\{ (n + 1/2) \pi \frac{(x)}{L} \frac{E}{F_3} \right\} \right]$$

$$+ C_i \exp\{\alpha x + (b - \gamma^2)T\}, \tag{20}$$

where

$$F_1 = (-1)^n (n + 1/2) \cos \left\{ (n + 1/2) \pi \frac{(x-L)}{(L)} \right\},$$

$$F_2 = \left\{ (n + 1/2)^2 \pi^2 D_0 + \gamma^2 R(L)^2 \right\},$$

$$F_3 = \left\{ (n + 1/2)^2 \pi^2 D_0 + bR(L)^2 \right\},$$

$$E = \exp \left\{ - \frac{(n + 1/2)^2 \pi^2 D_0}{R(L)^2} T \right\}, \quad b = \frac{D_0(\alpha - \beta)^2}{R}, \quad \beta = \frac{u_0}{2D_0},$$

$$\gamma^2 = \frac{u_0^2}{4RD_0} \quad \text{and} \quad T = \frac{1}{m} \{1 - \cos(mt)\}.$$

Case-II: Flux type boundary condition at the extreme end

In this case, we assumed flux type boundary condition at the outlet boundary. Mathematically flux type condition may be written as,

$$\frac{\partial c}{\partial x} = 0, \quad x=L, \quad t \geq 0 \quad (21)$$

The significance of boundary condition (21) is such that at the other end from the source, there is no solute exchange with the domain; i.e. mass flux is zero. Using condition (21) in place of (7b) and applying all transformation and taking Laplace transform, we get

$$\frac{d\bar{K}}{dx} = -\frac{u_0}{2D_0}\bar{K}; \quad x=L \quad (22)$$

Using condition (22) in place of (17b) with (17a) on general solution (18), the values of arbitrary constants are

$$C_1 = C_0 \left[\frac{2}{(p-\gamma^2)} - \frac{m}{(p-\gamma^2)^2} \right] \frac{M \cosh hM(L) + \beta \sin hM(L)}{M \cosh hM(L) - \beta \sin hM(L)} - \frac{C_i}{\left\{ p - \frac{D_0}{R}(\alpha - \beta)^2 \right\}} \frac{M \cosh hM(L) + \beta \sin hM(L)}{M \cosh hM(L) - \beta \sin hM(L)} \quad (23a)$$

and

$$C_2 = -C_0 \left[\frac{2}{(p-\gamma^2)} - \frac{m}{(p-\gamma^2)^2} \right] \frac{M \sin hM(L) + \beta \cosh hM(L)}{M \cosh hM(L) - \beta \sin hM(L)} - \frac{C_i \exp\{(\alpha - \beta)L\}}{\left\{ p - \frac{D_0}{R}(\alpha - \beta)^2 \right\}} \left[\frac{\alpha}{M \cosh M(L) - \beta \sin hM(L)} \right] + \frac{C_i}{\left\{ p - \frac{D_0}{R}(\alpha - \beta)^2 \right\}} \frac{M \sin hM(L) + \beta \cosh hM(L)}{M \cosh hM(L) - \beta \sin hM(L)} \quad (23b)$$

Taking inverse Laplace transform of equation (18) with values of constants (23a) and (23b), the analytical solution of advection-dispersion solute transport for periodic input condition in terms of $c(x,T)$, as

$$c(x,T) = C_1 F_1(x,T) + C_0 F_2(x,T) - C_0 F_3(x,T) \quad (24)$$

where

$$F_1(x,T) = \exp\{\alpha x - \gamma^2 t + bT\} - \exp\{\alpha L - \beta L + x\beta - \gamma^2 T + bT\} \left[\frac{\alpha \sinh\{(x)(\alpha - \beta)\}}{H_6} \right] - \exp\{x\beta - \gamma^2 T + bT\} E_3 + 2D_0 \sum_1^\infty \omega_n^2 \exp\left\{-\frac{D_0}{R} \omega_n^2 T\right\} \left[\exp\{\alpha L - \beta L + x\beta - \gamma^2 T\} \frac{\alpha \sin\{(x)\omega_n\}}{H_4} + \exp\{x\beta - \gamma^2 T\} E_2 \right] F_2(x,T) = \exp\{x\beta\} \left[E_1 - 2D_0 \exp\{-\gamma^2 T\} \sum_1^\infty \omega_n^2 \exp\left\{-\frac{D_0}{R} \omega_n^2 T\right\} E_{21} \right] F_3(x,T) = \exp\{x\beta\} \left[(-mT)E_1 + \frac{mR}{u_0\beta} \left\{ \frac{\cosh(L)\beta}{H_2} - \frac{\cosh(x-L)\beta}{H_2} - \beta L - \beta(x-L)E_1 \right\} \right] + 2D_0 \exp\{x\beta - \gamma^2 T\} \sum_1^\infty \left[\omega_n^2 \exp\left\{-\frac{D_0}{R} \omega_n^2 T\right\} \left\{ \frac{mR}{(\omega_n^2 D_0 + R\gamma^2)} \right\} E_{21} \right]$$

$$H_1 = \cosh(x-L)\beta - \sinh(x-L)\beta, H_2 = \cosh(L)\beta - \sinh(L)\beta, E_1 = \frac{H_1}{H_2},$$

$$H_3 = \omega_n \cos\{(x-L)\omega_n\} - \beta \sin\{(x-L)\omega_n\}$$

$$H_4 = (\omega_n^2 D_0 + Rb)\{\beta + (-\beta^2 + \omega_n^2)L\} \sin(L)\omega_n, E_2 = \frac{H_3}{H_4},$$

$$H_{41} = (\omega_n^2 D_0 + R\gamma^2)\{\beta + (-\beta^2 + \omega_n^2)L\} \sin(L)\omega_n, E_{21} = \frac{H_3}{H_{41}},$$

$$H_5 = (\alpha - \beta) \cosh\{(x-L)(\alpha - \beta)\} - \beta \sin\{(x-L)(\alpha - \beta)\},$$

$$H_6 = (\alpha - \beta) \cosh\{L(\alpha - \beta)\} - \beta \sinh\{L(\alpha - \beta)\}, E_3 = \frac{H_5}{H_6},$$

$$b = \frac{D_0(\alpha - \beta)^2}{R}, \beta = \frac{u_0}{4RD_0}, \omega_n \text{ is the positive root of the } [\omega_n \cot \omega_n L - \beta = 0], \text{ and } T = \frac{1}{m} \{1 - \cos(mt)\}.$$

Case-III: Concentration gradient at the extreme end is zero when the domain is semi-infinite

$$\text{i.e., } \frac{\partial c}{\partial x} = 0, \quad x \rightarrow \infty, \quad t \geq 0 \quad (25)$$

The physical significance of the semi-infinite condition (25) is that the exit boundary has negligible impact on transport. Using condition (25) in place of (7b) and applying all transformation and taking Laplace transform, we get

$$\frac{d\bar{K}}{dx} = -\frac{u_0}{2D_0}\bar{K}; \quad x \rightarrow \infty \quad (26)$$

Using condition (26) in place of (17b) with (17a) on general solution (18), the values of arbitrary constants are

$$C_1 = C_0 \left[\frac{2}{(p-\gamma^2)} - \frac{m}{(p-\gamma^2)^2} \right] - \frac{C_i}{(p-\gamma^2)^2} \text{ and } C_2 = 0 \quad (27)$$

Taking inverse Laplace transform of equation (18) with values of constants (27), the analytical solution of advection-diffusion solute transport for periodic input condition in terms of $c(x,T)$, as

$$C(x,T) = C_1 F_1(x,T) + C_0 F_2(x,T) \quad (28)$$

where

$$F_1(x,T) = \frac{1}{2} \exp\{\beta x + (b - \gamma^2)T\}$$

$$\left[\exp\left\{x\sqrt{\frac{bR}{D_0}}\right\} \operatorname{erfc}\left\{\frac{Rx + 2T\sqrt{D_0 b}}{2\sqrt{D_0 RT}}\right\} - \exp\{ax + (b - \gamma^2)T\} \right], \exp\left\{-x\sqrt{\frac{bR}{D_0}}\right\} \operatorname{erfc}\left\{\frac{Rx - 2T\sqrt{D_0 b}}{2\sqrt{D_0 RT}}\right\}$$

$$F_2(x,T) = \left[\operatorname{erfc}\left\{\frac{Rx - u_0 T}{2\sqrt{D_0 RT}}\right\} - \exp\left\{\frac{u_0 x}{D_0}\right\} \operatorname{erfc}\left\{\frac{Rx + u_0 T}{2\sqrt{D_0 RT}}\right\} \right]$$

$$- \frac{m}{2} \left[\left(T - \frac{x}{u_0}\right) \operatorname{erfc}\left\{\frac{Rx - u_0 T}{2\sqrt{D_0 RT}}\right\} - \left(T + \frac{x}{u_0}\right) \exp\left\{\frac{u_0 x}{D_0}\right\} \operatorname{erfc}\left\{\frac{Rx + u_0 T}{2\sqrt{D_0 RT}}\right\} \right],$$

$$b = \frac{D_0(\alpha - \beta)^2}{R}, \beta = \frac{u_0}{2D_0}, \gamma^2 = \frac{u_0^2}{4RD_0} \text{ and } T = \frac{1}{m} \{1 - \cos(mt)\}$$

Results and Discussion

Our goal is to understand the influence of the retardation and

periodic flow on the advection-dispersion equation solution given by solutions (20), (24) and (28). Model simulations are performed for periodic source configurations, and aquifers with horizontally finite and semi-infinite length. However, the simulations presented in this study are based on a continuous periodic source loading from point source. It was also shown that solute transport in subsurface porous media is significantly influenced by the aquifer boundary conditions. The utility of the proposed solution is demonstrated with numerical examples in two domains. All concentrations are conventionally expressed as dimensionless quantity $\frac{c}{C_0}$ with C_0 being equal to unity. The dimensionless concentration is investigated as a function of dimensional time and position.

The periodic behavior of concentration profile at inlet boundary i.e., x (km)=0.0 in time domain $0 \leq t$ (day) ≤ 105 for solution (28) is illustrated in Figure 1. The parameters are used; $R=1.30$, $m=0.15$, $C_i = 0.1$, $D_0=0.6$ (km²/day), $u_0=0.24$ (km/day) and $\alpha=0.024$.

Figures 2-7 are drawn for the obtained solutions (20) and (24) for finite domain $0 \leq x$ (km) ≤ 1 while Figures 8-10 are drawn for solution (28) of semi-infinite domain in finite region $0 \leq x$ (km) ≤ 10 . The numerical values of common parameters $D_0=0.6$ (km²/day), $u_0=0.24$ (km/day), $C_0=1.0$, $C_i=0.1$ and $\alpha=0.024$ are used for solution (20), (24) and (28).

The concentration behavior for solution (20) is depicted from Figures 2-4. Figure 2 shows the solute concentration profile at different time spans. The parameter used to demonstrate dimensionless concentration profiles are $R=1.05$, $m=0.1$. At the input boundary $x=0$, for the time t (day)=60, 125 and 190, the concentration values are 1.9601, 1.9978 and 1.9887 respectively. The nature of the concentrations is periodic at the inlet boundary. Periodic trends of concentration profiles remain unaltered in entire domain. The solute concentration decreases rapidly throughout the domain. Figure 3 illustrates the influence of various retardation factors $R=1.05$, 1.30, 1.55 on concentration profile with a fix unsteady parameter $m=0.1$ and time t (day)=125. At the inlet boundary change in retardations are almost negligible, means there is no significant role of retardation on concentration profile at the source boundary. Concentration values are higher for lower retardation up to certain position and emerges ineffective after distance x (km)=0.426 onwards. Figure 4 drawn for parameters t (day)=125, $R=1.55$ and various unsteady parameter $m=0.1$, 0.3 and 0.5. It reveals that concentration values are lower for smaller unsteady parameter and emerges ineffective near outlet boundary.

Figures 5-7 illustrate the concentration profile for solution (24).

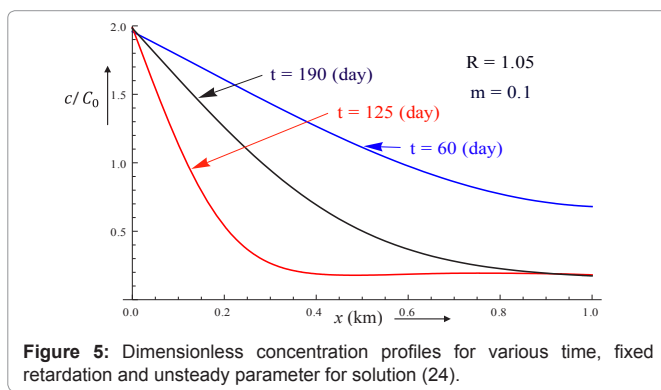
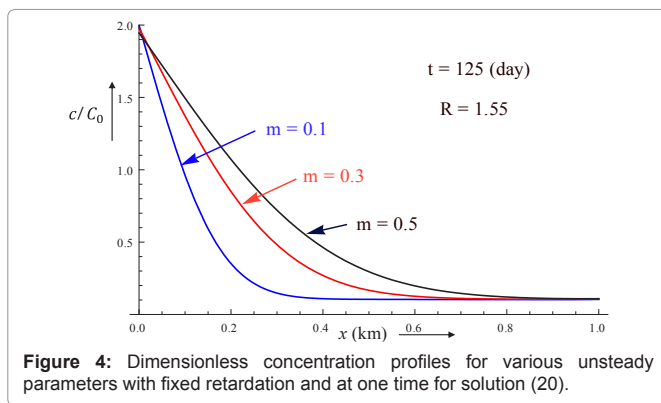
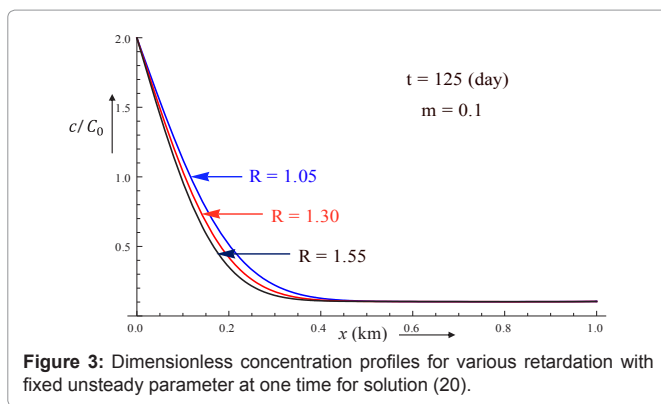
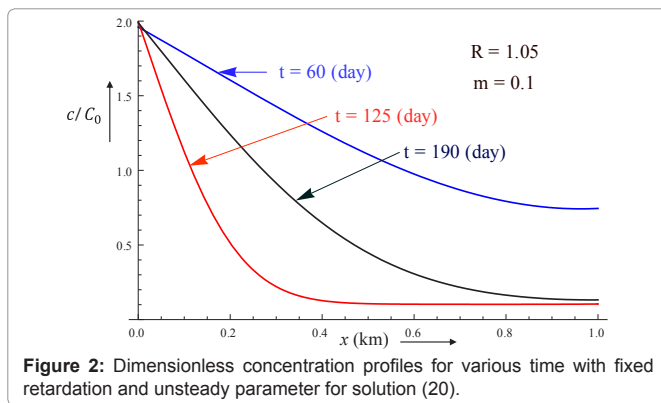
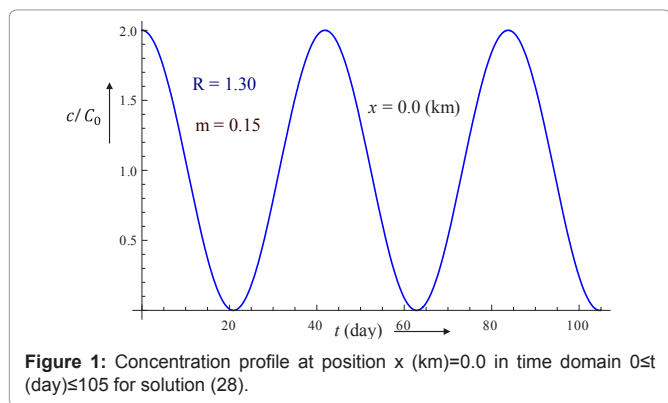


Figure 5 demonstrates the influence of concentration profiles on various times. The following parameters are used to demonstrate the effect on concentration profiles which are as retardation factor $R=1.05$, unsteady parameter $m=0.1$ and time t (day)=60, 125 and 190. The values of concentrations at the input boundary; i.e. $x=0$ are 1.9581, 1.9883, 1.9822. At the input boundary, nature of the concentration profiles is periodic. The concentration values decreases with position but the periodic trend maintained in entire domain. Figure 6 illustrates the effect of the retardation on concentration profiles in the domain. The parameters are used: $R=1.05, 1.30, 1.55, m=0.1$ and time t (day)=125. At the input boundary the values of concentration has no effect but it decreases rapidly up-to position $x=0.35$ and trends reverse towards outlet boundary. Figure 7 illustrates the effect of the unsteady parameter on concentration profiles in the domain. The parameters are used: $m=0.1, 0.3, 0.5, R=1.55$ and time t (day)=125. At the inlet boundary the concentration values varying periodically but with position its value rapidly decreases with decreasing unsteady parameter.

Figures 8-10 are drawn for solution (28). The concentration profiles for various times are shown in Figure 8. The parameters used to demonstrate the effect of concentrations are: $m=0.15, R=1.30$ and time t (day)=15, 45, 75. Initially the concentration levels are varying periodically with time. The concentration values emerge after position x (km)=4.5. The concentration profiles for various retardations are shown in Figure 9. The parameters used to illustrate the concentration behavior are: $m=0.15$, time t (day)=45 and $R=1.30, 1.60$, and 1.90 . At input boundary the concentration values are approximately same for various retardation factors. The concentration values rapidly decreases for various retardation and emerges at position x (km) =4.5 and beyond it the significance are ineffective. Figure 10

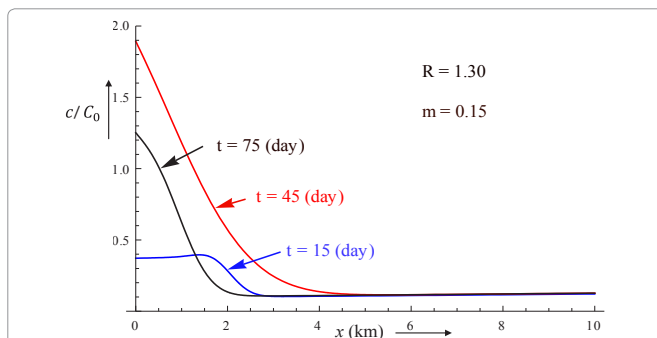


Figure 8: Dimensionless concentration profiles for various time, fixed retardation and unsteady parameter for solution (28).

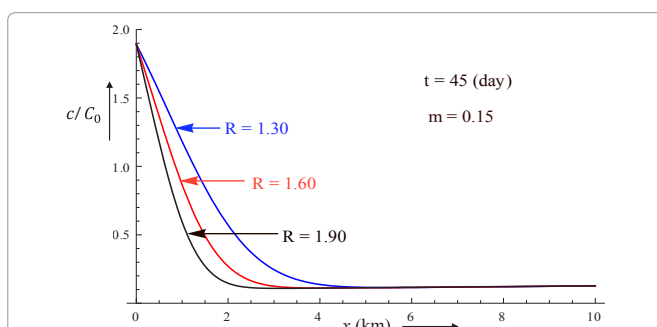


Figure 9: Dimensionless concentration profiles for various retardation with fixed unsteady parameter at one time for solution (28).

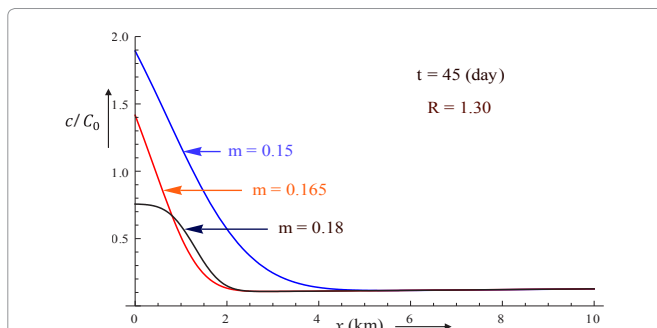


Figure 10: Dimensionless concentration profiles for various unsteady parameter with fixed retardation and at one time for solution (28).

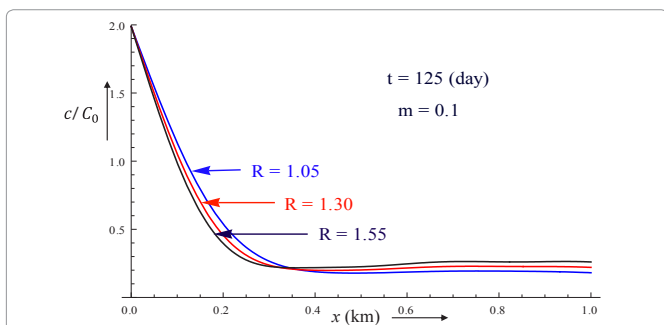


Figure 6: Dimensionless concentration profiles for various retardation with fixed unsteady parameter at one time for solution (24).

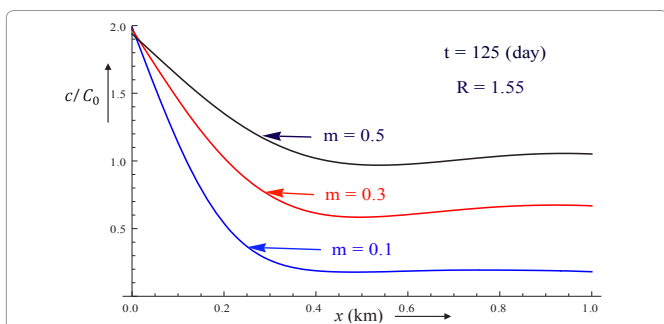


Figure 7: Dimensionless concentration profiles for various unsteady parameters with fixed retardation and at one time for solution (24).

illustrates the effect of unsteady parameter on concentration profile. At the inlet boundary unsteady parameter plays significant effect on concentration profile. Near source boundary the concentration level rapidly decreases with increasing unsteady parameter and emerges at position x (km)=4.5 and towards outlet boundary its effect is insignificant. The parameters used to demonstrate the effects of concentration are: $m=0.15, 0.165, 0.18$, time t (day)=45 and $R=1.30$.

In rainy season the aquifers natural recharge changes with distance and the recharge acuteness. It is well known fact that the recharge rates significantly influence the tracer distribution and may explain the time dependence of dispersion parameter. Knowledge of the time-dependent behavior of solutes in the sub-surface is of the interest for many problems where the concentration is observed or needs to be predicted at fixed position. The periodic solute mass

transport model in porous medium remains a key issue in the area of hydrogeology & hydrologic engineering because various contaminants frequently enter the aquifers, either by known activities or by accident, and affects the environment. The application of analytical solution reveals that the solute mass transport process at the test site obeys the time dependent dispersion model and validates the numerical and experimental solutions.

Conclusion

The effect and influence of periodic time dependent contaminant injection into an aquifer is considered. The water table is assumed horizontal and the porous medium comprising the aquifer is considered homogeneous, isotropic and adsorbing nature. Laplace transformation technique is applied on this model with consideration of the effects on contaminant transport of such parameters as groundwater flow, adsorption coefficient, and the unsteady parameter. The simple time-dependent inlet condition consisting periodic input function is considered to demonstrate the applicability of the analytical solution for development of the analytical solution. The unsteady parameter m has strong impact on the transport of contaminant. The solutions are useful for analyzing the possible prevention of the spread of poor-quality water by a flow of fresh water. As the characteristics of solutes are conservative and the subsurface flow varied periodically with time. The temporal variations of solutes are influenced by the different periodic process.

Acknowledgement

This work is carried out under Post Doctoral Fellowship programme of author Dilip Kumar Jaiswal and gratefully acknowledges the financial assistance in the form of UGC-Dr. D. S. Kothari Post Doctoral Fellowship, New Delhi, India.

References


1. Ferris JG (1963) Cyclic water-level fluctuations as a basis for determining aquifer transmissibility. In: 'Methods of determining permeability, transmissibility and drawdown, compiled by Bentall Ray'. US Geol Survey Water-Supply Paper: 305-318.
2. Kumar N (1983) Dispersion of Pollutants in Semi-Infinite Porous Media with Unsteady Velocity Distribution. *Nordic Hydrol* 14: 167-178.
3. Rao NH, Sarma PBS (1984) Recharge to aquifers with mixed boundaries. *J Hydrol* 74: 43-51.
4. Kinzelbach W, Ackerer P (1986) Modélisation du transport de contaminant dans un champ d'écoulement non-permanent. *Hydrogeology* 2: 197-205.
5. Gelhar LW (1993) *Stochastic Subsurface Hydrology*. Prentice Hall, Englewood Cliffs, NJ.
6. Goode DJ, Konikow LF (1990) Apparent Dispersion in Transient Groundwater Flow. *Water Resour Res* 26: 2339-2351.
7. Webster IT, Taylor JH (1992) Rotational dispersion in porous media due to fluctuating flows. *Water Resour Res* 28: 109-119.
8. Li L, Barry DA, Jeng DS (2001) Tidal fluctuations in a leaky confined aquifer: dynamic effects of an overlying phreatic aquifer. *Water Resour Res* 37: 1095-1098.
9. Jeng DS, Li L, Barry DA (2002) Analytical solution for tidal propagation in a coupled semi-confined/phreatic coastal aquifer. *Adv Water Resour* 25: 577-584.
10. Sun DM, Zhu YM, Zhang MJ (2007) Unsteady seepage problems due to water level fluctuation. *Journal of Tian Jin University* 40: 779-785.
11. Jaiswal DK, Kumar A, Kumar N, Yadav RR (2009) Analytical solutions for temporally and spatially dependent solute dispersion of pulse type input concentration in one-dimensional semi-infinite media. *Journal of Hydro-environment Research* 2: 254-263.
12. Jaiswal DK, Kumar A, Kumar N, Singh MK (2011) Solute Transport along Temporally and Spatially Dependent Flows through Horizontal Semi-Infinite Media: Dispersion Proportional to Square of Velocity. *J Hydrol Eng (ASCE)* 16: 228-238.
13. Kumar A, Jaiswal DK, Kumar N (2009) Analytical solutions of one-dimensional advection-diffusion equation with variable coefficients in a finite domain. *J Earth Syst Sci* 118: 539-549.
14. Yadav RR, Jaiswal DK, Yadav HK, Gulrana (2010) One-Dimensional Temporally Dependent Advection-Dispersion Equation In Porous Media: Analytical Solution. *Natural Resource Modeling* 23: 521-539.
15. Pérez Guerrero JS, Pimentel LCG, Skaggs TH, van Genuchten MTh (2009) Analytical solution of the advection-diffusion transport equation using a change-of-variable and integral transform technique. *Int J Heat Mass Tran* 52: 3297-3304.
16. Chen JS, Liu CW (2011) Generalized analytical solution for advection-dispersion equation in finite spatial domain with arbitrary time-dependent inlet boundary condition. *Hydrol Earth Syst Sci* 8: 4099-4120.
17. Freeze RA, Cherry JA (1979) *Groundwater*. Prentice Hall, Inc.
18. Bear J (1972) *Dynamics of Fluid in Porous Media*. Dover Publ, New York, USA.
19. Lapidus L, Amundson NR (1952) Mathematics of Adsorption in Beds. VI. The Effect of Longitudinal Diffusion in Ion Exchange and Chromatographic Columns. *J Phys Chem* 56: 984-988.
20. Yim CS, Mohsen MFN (1992) Simulation of Tidal Effects on Contaminant Transport in Porous Media. *Groundwater* 30: 78-86.
21. Crank J (1975) *The Mathematics of Diffusion*, (2nd edn). Oxford Univ Press Inc., New York, USA.

Author Affiliation

Top

¹Department of Mathematics and Astronomy, Lucknow University, Lucknow-226007, India

Submit your next manuscript and get advantages of SciTechnol submissions

- ❖ 50 Journals
- ❖ 21 Day rapid review process
- ❖ 1000 Editorial team
- ❖ 2 Million readers
- ❖ More than 5000 
- ❖ Publication immediately after acceptance
- ❖ Quality and quick editorial, review processing

Submit your next manuscript at • www.scitechnol.com/submission