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Editorial

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Expansion Perturbation Method for Nonlinear Vibrations

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Abstract

This paper originates from and is motivated by a recently published manuscript, where the influence of various support types within the nonlinear vibrations of beams is analyzed. During the event of this latter paper, the authors looked for a proof of the simple Expansion Perturbation Method (SEPM), not reaching any manuscript containing a correct and rigorous definition and proof of the tactic. The most objective of this paper is to supply a mathematical formalism of SEPM.

Currently, an outsized number of nonlinear vibration problems in Engineering are solved by the Nonlinear Finite Element Method. However, in many cases, it's necessary to seek out an analytical solution so as to raised understand the contribution of forces, masses or geometries. within the process of checking out an analytical solution, hypotheses, simplifications and linearizations are raised, which usually cause approximations of the precise analytical solutions. Traditionally, nonlinear problems are solved by perturbations methods so as to eliminate the generated secular terms. consistent with these techniques, the answer is represented by a couple of terms of an expansion, usually no quite two or three terms. Therefore, the deviation between the approximate analytical solution and therefore the refore the exact analytical solution depends on the amount of selected expansion terms and the amplitude of the vibration.

There are typically three perturbation methods for approaching the answer of weakly nonlinear vibrations: the tactic of Multiple Scales, the tactic of Harmonic Balance and therefore the Method of Averaging. there's a fourth method, simpler to use than the previous three but far more imprecise, called the simple Expansion Perturbation Method (SEPM). This perturbation method applied to weakly nonlinear vibrations doesn't usually yield to correct solutions. the looks of secular terms, i.e., terms of the shape $t \sin(t + \phi 0)$, results in incorrect solutions thanks to, among other physical reasons, the unbounded growth of the term with time t. In Theorem 3 and Corollary 4, we prove that, if the terms of the expansion are uniformly bounded, then the SEPM results in an accurate solution of a weakly nonlinear vibration.

There are other perturbation methods like the tactic of Strained Parameters (also referred to as Lindstedt-Poincaré Method) and therefore the Naive Singular Perturbation Method. Both methods also can be considering expansion methods, within the sense that the answer of the nonlinear equation is calculated by means of an expansion series, love it occurs with SEPM. the most difference between the SEPM and therefore the previous two methods relies on the way that the expansion terms are computed. On the opposite hand, as mentioned above, we offer a proof that assures that, under uniform boundedness of the expansion terms, the expansion series within the SEPM converges to the answer we've conveyed an enquiry and that we have found no mathematical theorems or formalism for the tactic of Strained Parameters or the Naive Singular Perturbation Method. Both methods are described by means of examples and successfully applied in many situations. during this direction, we refer the reader to many works by Van Groesen and his students Karjanto and Cahyono, who accomplished more successful applications of perturbartion methods in several settings, like wave modelling.

In 1998, Liao provided a replacement analytic technique, called Homotopy Analysis Method (HAM), which differs from perturbation methods within the essential incontrovertible fact that the validity of HAM is independent on whether or not there exist small parameters in considered nonlinear equations. Therefore, HAM may be a powerful tool to affect strongly nonlinear problems. As we'll show throughout this manuscript, the existence of small parameters is crucial towards the convergence of SEPM. In fact, SEPM may be a powerful tool for weakly nonlinear vibrations during which the norm of the expansion terms $(\theta i)i \geq 0$ are often uniformly controlled. it's worth mentioning that we offer a proof for the validity of SEPM within the previously mentioned weakly nonlinear vibrations, whereas no proof is given for a general validation of HAM. However, Liao applied HAM successfully in several situations like boundary element methods, the laminar viscous flow over a semi-infinite flat plate, and nonlinear oscillations. HAM is compared with Euler transformations.

Many authors have investigated the SEPM and developed formulations. However, a rigorous mathematical treatment has not been administered . Therefore, it's necessary to define the limit of the SEPM so as to supply mathematical formalism and contribute to the advancement of this method. A Banach algebra of differentiable functions endowed with an extended supremum norm has been utilized in this work. Many theorems, corollaries and lemmas are formulated with the aim to assure the steadiness of the solutions found with this method. additionally , all the methodology used has been applied to the instance of the famous pendulum.

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