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Formalizing Calculus while not Limit Theory in Coq

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Abstract

The rationalisation of the basic theory can contribute to the event of huge comes. During this paper, we tend to gift the rationalisation in Coq of calculus while not limit theory. The idea aims to found a replacement variety of calculus additional simply however strictly. This theory as Associate in Nursing innovation differs from ancient calculus however is equivalent and additional clear. First, the definition of the difference-quotient operation is given intuitively from the physical facts. Further, conditions area unit other to that to urge the spinoff, and outline the integral by the axiomatization. Then some vital conclusions in calculus like the Newton–Leibniz formula and therefore the Taylor formula will be formally verified. This shows that this theory will be freelance of limit theory, and any proof doesn't involve imaginary number completeness. This work will facilitate learners to review calculus and lay the muse for several applications.

Keywords: Calculus; Difference-quotient management function; Coq; Formalization; Limit theory

Introduction

As proof assistants Coq, Isabelle and HOL and then on develop, formal arithmetic has achieved goodly progress. In 2005, Gonthier and Werner proven the famed "Four Color theorem" in a very absolutely formal means. Further, Gonthier given the formal proof of the "Odd Order Theorem" in 2012. In 2017, Hales et al. formalized "Kepler Conjecture" to verify his previous proof. In 2019, Cruz-Filipe et al. completed the rationalisation of the "Boolean philosopher Triples Problem" , that checked the event of Marijn et al. Those achievements build formal arithmetic recognized and advocated by mathematicians. Moreover, the formalizations need mathematical understanding and familiarity with tools, and therefore the use of developed libraries in a very form of proof assistants. Formalizing the basic theory can contribute to the event of huge comes.

As one of the milestone accomplishments in arithmetic history, calculus was based over three hundred years past. It greatly promoted the event of arithmetic and alternative scientific fields, and resolved several issues in sensible engineering. Newton and Leibniz initial created calculus, that settled several historical mathematical issues at that point and created a profound influence. The conception of obscure small, however, it absolutely was questioned and this caused the second mathematical crisis.

Lagrange endeavored to ascertain the entire calculus theory on Taylor formula to avoid handling Newton's "fluxion", and Leibniz's "infinitesimal", however the convergence of infinite series still couldn't avoid the conception of limit. Till the 19th century, the progress of study diode mathematicians to ascertain a rigorous limits theory. The landmark work was chiefly conducted by Cauchy, metropolis and Weierstrass, and outlined the limit strictly by "e-N". This theory basically solves the second mathematical crisis and has become a solid foundation of recent analysis. The difficult ideas and reasoning, however, it's exhausting for a few beginners to soak up and perceive.

In order to alter learners to review calculus theory with less time and energy, many students perpetually try to ascertain a replacement theory. By researchingcalculus full, this idea became a reality and goodly progress has been created. "The limit theory isn't a necessity for learning calculus, and high school students with information of operate will perceive the bulk of calculus topics". Calculus while not limit theory will facilitate beginners to be told additional quickly, additional simply, and additional effectively, by the link between calculus ideas and therefore the physical world, to speedily master the foremost crucial skills in calculus and settle some issues in arithmetic and reality.

The calculus while not limit theory will be based strictly and comprehensibly. Moreover, most propositions whose proof has to involve completeness, continuity and limit in ancient calculus will be proven directly during this theory. This doesn't mean, however, that limit theory ought to ne'er be learned. Calculus looking forward to imaginary number theory and limit theory is doubtless a qualitative leap within the history of arithmetic. Actually, it's useful to be told limit theory to know some calculus information far better and far additional deeply. Thus, formalizing calculus while not limit theory has specific significance for teaching and analysis.

Our rationalisation attracts lessons from all the previous works and kinds out a additional complete and systematic version. This work relies on the formal system of Landau's "Foundations of Analysis". there's little or no propaedeutic information needed during this treatise. ranging from the Peano axioms, the entire theory of the amount system from natural numbers to advanced numbers is given successively. As Associate in Nursing application of our system, we tend to formally completed the cyclic proof of eight completeness theorems and therefore the proof of properties of never-ending operate on closed intervals. It ought to be noted that we tend to solely use the contents before Section four.4 that don't involve completeness (Dedekind basic theorem) nonetheless.

In this work, the conception of difference-quotient operation is initial introduced from motion law. Next, the elementary definitions of uniform spinoff and powerful spinoff will be naturally obtained.Then, the integral system and integral area unit given by axiomatization. At constant time, we tend to discuss the link between them. what is more, we tend to gift the connected conception of upper order spinoff. At last, some vital theorems in calculus will be directly proven, and this means that this theory doesn't place confidence in imaginary number completeness and limit theory. On the one hand, each proof is verified by Coq to indicate rigor and correctness. On the opposite hand, we tend to form up for missing proof details to create it additional complete. additionally, there area unit some innovations of formal ways within the proof method. The proofs area unit checked formally

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mistreatment the Coq proof assistant, and therefore the machine proving progress is rigorous and reliable.

The paper is organized in the following way. Section 2 is dedicated to related work. Section 3 briefly states the necessary background needed for understanding this work. Section 4 introduces some definitions and consequences concerning them needed in establishing the theory. Section 5 presents the formalization of calculus without limit theory. Section 6 describes the definition of higher order derivative and related properties. Section 7 discusses the proof of critical theorems in calculus. Finally, we draw our conclusions and discuss some potential further work in Section 8.

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