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Macroscopic Quantum-Mechanical Scattering. Coherent Scattering of Neutrinos

Apostol M*

Department of Theoretical Physics, Institute of Atomic Physics, Magurele-Bucharest MG-6, POBox MG-35, Romania

***Corresponding author:** Apostol M, Department of Theoretical Physics, Institute of Atomic Physics, Magurele-Bucharest MG-6, POBox MG-35, Romania; E-mail: apoma@theory.nipne.ro

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Abstract

In certain conditions a macroscopic quantum-mechanical scattering may occur, which may lead to a coherent cross-section on a macroscopic scale in a mono-crystal. The conditions are satisfied by neutrinos, but not satisfied by other projectiles, with a higher cross-section. This may explain Weber-type experiments of neutrino detection by a perfect, stiff sapphire mono-crystal. The occurrence of coherence domains for quantum-mechanical scattering and classical diffraction is analyzed, and the force exerted upon a macroscopic target is estimated. It is concluded that neutrinos exhibit a distinctive feature in this respect, due precisely to their very small cross-section.

Keywords: Neutrino scattering; Coherent scattering; Macroscopic coherence; Neutrino detection

Introduction

In two papers published in 1985 and 1988 Weber claimed that neutrinos (antineutrinos), from various sources like tritium, nuclear reactors and Sun, could be detected by their coherent scattering by a perfect, stiff, sapphire mono-crystal with a high Debye temperature (mounted on a torsion balance and equilibrated by a lead dummy)[1,2]. The coherent cross-section would be $\sigma=N^2\sigma_0$, where N is the number of unit cells in the target and σ_0 is the cross-section of a single unit cell (particle, e.g., atomic nucleus). Such a highly enhanced cross-section $\sim N^2$ would give rise to a measurable force upon a torsion balance. Weber's claims have been criticized both on theoretical and experimental grounds, the main objection being that the form-factor would reduce appreciably the cross-section, and, on the other hand, such a coherence effect is not observed in X, gamma rays or neutron scattering [3-7]. A discussion of the theoretical objections and negative experiments was given by Nicolescu, who presented a positive experiment; indeed, an experiment by Cruceru et al., exists, which confirms Weber's prediction for solar neutrinos. The problem is still controversial. We show in this paper that a coherent scattering of neutrinos may appear in the conditions formulated by Weber and company, as a consequence of a quantum-mechanical treatment of the crystal as a whole (a macroscopic quantum-mechanical scattering). This is a distinctive condition of the neutrino scattering, which is not fulfilled by other projectiles, with a higher cross-section. The main reason for such a behavior is precisely the extremely small cross-section $\sigma 0$ (of the order 10^{-44} cm^2) of the neutrinos [8-10].

For $\sigma_0=10^{-44}$ cm^2 and $N=10^{22}$ (less than 0.1 mol) the coherent cross-section is $\sigma=1$ cm^2. For a neutrino flux density $\Phi=10^{12}$/cm^2 s. The time between two collisions is $\tau=1/\Phi\sigma=10^{-12}$ s. On the other hand, an atom in thermal equilibrium at room temperature has a velocity of the order $v=10^4$ cm/s. In an elementary act of collision the atom is perturbed from its equilibrium state and receives a momentum of the order $p=E/c$, on average, where E is the energy of the neutrino projectile (and $c=3\times 10^{10}$ cm/s is the speed of light). For E=1MeV the momentum transfer is of the order $p=5\times 10^{-17}$g cm/s. Consequently, the energy perturbation of the atom is of the order $\Delta E=p^2/M=2.5\times 10^{-11}$ erg for $M=10^5$ m$_e$, where m$_e \backsimeq 10^{-27}$ g is the electron mass ($vp=5\times 10^{-13}$ erg $\ll p^2/2M$). The time needed for this atom to recover its equilibrium is of the order $\Delta t_{eq}=\hbar/\Delta E \backsimeq 4\times 10^{-17}$s (where $\hbar \backsimeq 10^{-27}$ erg.s is Planck's constant). We can see that $\tau \gg \Delta t_{eq}$. It follows that the atoms recover quickly their equilibrium state between two successive collisions, and the incident neutrino beam sees the crystal as a whole. Therefore, we need to adopt a quantum-mechanical treatment for the entire crystal. It is worth noting that if the cross-section increases to, say, $\sigma_0=10^{-24}$ cm^2, as for X-, gamma rays or neutrons, the "collision" time decreases to $\tau=10^{-32}$ s, which is much shorter than the equilibrium time Δt_{eq}, all the other conditions remaining the same. In that case the incident projectile beam sees the crystal as consisting of distinct, independent atoms, such that a coherent scattering ($\sigma=N^2\sigma_0$) for the entire crystal is not possible. The particularity of a coherent scattering suffered by neutrinos in the whole crystal resides precisely in their extremely small cross-section σ_0. On the other hand, a single-particle cross-section $\sigma_0=10^{-24}$ cm^2 increases considerably the total cross-section, such that we need to reconsider the scattering in this case [11].

Description

Macroscopic quantum-mechanical scattering: Let us assume a macroscopic target consisting of $N \gg 1$ identical "atoms" (atomic nuclei, molecules, unit cells in a crystal). The interaction with an incident beam of particles can be written as

$$H = a^3 h(\xi) \sum_{i=1}^{N} \delta(\boldsymbol{r} - \boldsymbol{r}_i) \qquad (1)$$

Where a is the range of the single-particle interaction h (ξ), ξ denotes the internal coordinates of the atoms and r_i are the atomic positions. The time between two successive collisions is $\tau=1/\Phi\sigma$, where Φ the incident is flux density and σ is the total cross-section. In an elementary act of collision an atom receives a momentum of the order of the momentum p of the incident particle. The atom has a thermal velocity

$$v \simeq \sqrt{T/M}$$

Where, T is the temperature and M is the mass of the atom. The atomic motion is perturbed by an energy of the order $\Delta E=p^2/M+vp$, so it needs

a time $\Delta t_{eq} \simeq \hbar/\Delta E$ E to recover its equilibrium. Let us assume

$$\tau > \Delta t_{eq} \qquad (2)$$

then, the incident particles see the macroscopic target as a whole, and we need to work with the wave function of the whole, macroscopic, target. A perfect mono-crystal suffers two kinds of motion. One kind is the motion of the crystal as a whole, where all the laticial positions of the atoms $\mathbf{r}_i^0$ move by the same distance. The wave function corresponding to this motion is

$$\Phi_{\boldsymbol{K}}(\xi;\boldsymbol{r}) = \sum_{i=1}^{N} e^{i\boldsymbol{K}\boldsymbol{r}_i^0}\varphi(\xi;\boldsymbol{r}-\boldsymbol{r}_i^0) \ , \qquad (3)$$

Where K is the quasi-wave vector of the crystal (quasi-momentum ħK) and

$$\varphi(\xi;\boldsymbol{r}-\boldsymbol{r}_i^0)$$

are (orthonormal) wave functions localized on the positions $\mathbf{r}_i^0$. We can see that the wave function Φ_K has the translational symmetry of the crystal. We call Φ_K a coherent wave function. The other type of motion of the crystal is the thermal motion with atomic displacements

$$\boldsymbol{v}_i \ (\boldsymbol{r}_i = \boldsymbol{r}_i^0 + \boldsymbol{v}_i);$$

the corresponding wave function is

$$\psi(\xi;\boldsymbol{r}) = \sum_{i=1}^{N} e^{i\chi_i}\varphi(\xi;\boldsymbol{r}-\boldsymbol{r}_i^0-\boldsymbol{v}_i) \ , \qquad (4)$$

where $e^{i\chi i}$ are random phase factors; we call ψ an incoherent wave function. Both these wave functions are normalized to N; they are orthogonal to each other. The wave function of the crystal is

$$\Psi_{\boldsymbol{K}} = \sqrt{1-g^2}\Phi_{\boldsymbol{K}} + g\psi \ , \qquad (5)$$

where g is a weight coefficient. This coefficient is proportional to the square root of the relative number of vibration states of the crystal, properly normalized. This relative number of phonon states is proportional to

$$I = \frac{1}{\omega_D^3}\int_0^{\omega_D} d\omega \frac{\omega^2}{e^{\hbar\omega/T}-1} \ , \qquad (6)$$

where ω_D is the Debye frequency and ω is the phonon frequency; the Debye temperature is $\Theta = \hbar\omega_D$. For $T/\Theta \ll 1$ the integral is $I \simeq 2.4(T/\Theta)^3$, while for $T/\Theta \gg 1$ the integral is $I \simeq \frac{1}{2}T/\Theta$. A normalized expression for the relative number of states is $4.8(T/\Theta)^2$ for $T/\Theta \ll 1$ and 1 for $T/\Theta \gg 1$. An interpolation formula is $I \simeq 4.8(T/\omega_D)^2/\left[1+4.8(T/\omega_D)^2\right]$, such that we can take for the weight coefficient

$$g \simeq \frac{2.2(T/\Theta)}{\left[1+4.8(T/\Theta)^2\right]^{1/2}} \ . \qquad (7)$$

We can see that for high Debye temperatures the main contribution to Ψ_K comes from the coherent wave function (g≪1), while for low Debye temperatures the main contribution comes from the incoherent field (g→1).

Let us assume a wave function

$$\frac{1}{\sqrt{V}}e^{i\boldsymbol{k}\boldsymbol{r}}$$

for an incident particle, and an initial (i) wave function

$$\frac{1}{\sqrt{V}}e^{i\boldsymbol{k}\boldsymbol{r}}\Psi_{\boldsymbol{K}}$$

where V denotes the volume. The normalization to unity of this wave function requires a factor

$$\sqrt{a^3}$$

in the wave fuctions φ, such that the scalar product is

$$\left\langle \varphi(\xi;\boldsymbol{r}_j-\boldsymbol{r}_i), \varphi(\xi;\boldsymbol{r}_j-\boldsymbol{r}_i)\right\rangle^2 = \delta_{ij}$$

The matrix elements of the interaction between the wave functions Φ_K and ψ are zero. The matrix elements of the interaction between two wave functions Φ_K and $\Phi_{K'}$ (coherent scattering) lead to the momentum conservation

$$H_{fi} \sim \sum_{i=1}^{N} e^{i(K-K')r_i^0}e^{i(k-k')r_i^0} = N\delta_{\boldsymbol{K}'+\boldsymbol{k}',\boldsymbol{K}+\boldsymbol{k}} \ , \qquad (8)$$

where k′, K′ are the wave vectors of the final state (f). We can see that the difference in momentum of the incident particle is taken by the crystal, which moves as a whole. According to equation (8) the coherent cross-section is

$$\sigma_{coh} = N^2\sigma_0 \ , \qquad (9)$$

Where σ_0 is the single-particle cross-section. A similar calculation for the incoherent matrix elements (wave functions ψ) leads to

$$H_{fi} \sim \sum_{i=1}^{N} e^{i(k-k')r_i^0}\left(1+i(\boldsymbol{k}-\boldsymbol{k}')\boldsymbol{v}_i+...\right) \ , \qquad (10)$$

Where

$$H_{fi} \sim \sqrt{N}\left(\boldsymbol{q}\boldsymbol{v}_q\right)\delta_{k',k+q} \ . \qquad (12)$$

We can see that the difference in momentum of the incident particle is taken by phonons; the incoherent scattering excites phonons. In addition, the incoherent cross-section is proportional to N. For the cross-section we need to average $(qv_q)^2$ over the thermal states. The maximum value of this average is of the order T/Mc_s^2 (at room temperature), where c_s is the mean phonon velocity (of the order 10^6 cm/s). This thermal factor is reminiscent of the Debye-Waller factor (and the diffuse scattering). The total incoherent cross-section can be written as

$$\sigma_{incoh} = N\sigma_0^{ph} \quad , \tag{13}$$

Where the single-particle cross-section σ_0^{ph}, arising from phonons, is smaller than σ_0 by the thermal factor.

The total cross-section of the crystal is

$$\sigma = \sqrt{1-g^2}\sigma_{coh} + g^2\sigma_{incoh} \ . \tag{14}$$

The incoherent cross-section is extremely small in comparison with the coherent cross-section. For atoms placed randomly (like in amorphous solids, liquids, etc) the coherent wave function Φ_K is absent and the weight coefficient is g=1.

Neutrino scattering

We adopt $N \simeq 10^{22}$ for the number of unit cells in the sapphire monocrystal ($\simeq$ 24 g, density 4 g/cm^3) used in Weber's experiments and other similar experiments (the volume of the unit cell of sapphire is large, $\simeq 10^3$Å^3). Making use of $\sigma_0=10^{-44}$ cm^2 we get a coherent cross-section $\sigma_{coh} \simeq 1$ cm^2. For a Debye temperature $\Theta=10^3$K the weight factor at room temperature is

$$\sqrt{1-g^2} \simeq 0.7.$$

The total coherent cross section is $\sigma \simeq 0.7$ cm^2. We note that this cross-section is smaller than the cross-sectional area of the crystal. For a flux density $\Phi=10^{12}$/cm$^2 \cdot$ s the collision time is $\tau=1/\Phi\sigma_{coh} \simeq 10^{-12}$ s. At room temperature the thermal velocity of an atom is $v \simeq 10^4$ cm/s. For a neutrino energy E=1 MeV the momentum is $p=E/c \simeq 5 \cdot 10^{-17}$g $\cdot$ cm/s. The perturbation energy is $\Delta E=p^2/M \simeq 2.5 \times 10^{-11}$ erg (for an atomic mass $M=10^5$ m_e, where m_e is the electron mass); the contribution $E=vp \simeq 5 \times 10^{-13}$ erg is much smaller. The equilibrium time is of the order $t_{eq}=\hbar/E \simeq 4 \times 10^{-17}$ s. Since $\tau \gg$ teq the macroscopic quantum-scattering described above applies.

The force acting upon the target in the forward direction is $F=\Phi\sigma p \simeq 3.5 \times 10^{-5}$ dyn. This is a measurable force. We note that it is sensitive to the values of the input parameters. For instance, a Debye temperature $\Theta=100$ K leads to a weight coefficient

$$\sqrt{1-g^2} \simeq 0.15$$

and a weaker force by a factor $\simeq 5$. Also, for an amorphous solid, although the conditions of a quantum-mechanical scattering may be fulfilled, the force is extremely weak, as a consequence of the very small incoherent cross-section.

For solar neutrinos ($E \simeq 300$ keV) the single-particle cross-section may preserve its value, but the flux density is smaller ($\Phi \simeq 10^{11}$/cm$^2 \cdot$ s). The conditions of a coherent scattering are preserved, but the force may be diminished by a factor $\simeq 10^{-1}$ ($\simeq 10^{-6}$ dyn). Also, a decrease may appear for tritium neutrinos (E=10keV, $\Phi \simeq 10^{14}$/cm$^2 \cdot$ s), though a higher σ_0 or a slightly greater number of unit cells N may compensate the decrease (while preserving the conditions of coherent scattering). We note that a large number of atoms in the unit cell, as for a sapphire crystal, may increase the single-particle cross-section σ_0. We conclude that Weber-type experiments could exhibit measurable force acting upon a sapphire crystal.

Other projectiles

Coherence domains: We adopt the value $\sigma_0=10^{-24}$cm^2 for other types of projectiles (like X, gamma rays or neutrons). A coherent cross-section would be much larger than the cross-sectional area of the crystal. The crystal responds to this unphysical situation by developing coherence domains. Let us assume that nd uncorrelated domains exist in the crystal, each with Nd unit cells (as a mean size), such that nd=N/Nd. By a formal analogy with the high-purity crystals we use the fraction f=1/Nd. This fraction varies between f=1/N, when we have only one domain, *i.e.* the whole target, and f=1, when the whole target is fragmented in "atomic" domains.

The scattering amplitude can be written as

$$S = \sum_{a=1}^{n_d} e^{i\chi_a} (H_{fi})_a \quad , \tag{15}$$

Where

$$e^{i\chi_a}$$

are random phase factors. By averaging the squared scattering amplitude over the phase factors, we get

$$|S|^2 = \sum_a \left|(H_{fi})_a\right|^2$$

Such that the cross-section becomes

$$\overline{\sigma} = n_d\sigma_d \quad , \tag{16}$$

Where σ_d is the cross-section of a domain. According to this equation, the coherent cross-section $\sigma_{coh}=\sigma_0N^2$ is reduced by the coherence domains to

$$\overline{\sigma} = \sigma_0 n_d N_d^2 = \frac{\sigma_0 N}{f} \ ; \tag{17}$$

we can see that this formula gives the total coherent cross-section (σ_0N^2) for f=1/N and an incoherent cross-section (σ_0N) for f=1. In this latter case σ_0 should be replaced by

$$\sigma_0^{ph}$$

(a similar procedure leaves the incoherent cross section arising from phonons unchanged,

$$\overline{\sigma}_{incoh} = n_d N_d \sigma_0^{ph} = N\sigma_0^{ph} = \sigma_{incoh}).$$

In order to have a quantum-mechanical scattering the conditions $\bar{\tau}$ t_{eq} and $\bar{\sigma}$ A should be satisfied, where A is the cross-sectional area of the target; these conditions lead to

$$f > \frac{\hbar\Phi}{\Delta E}\sigma_0 N \ , \ f > \frac{\sigma_0 N}{A} \tag{18}$$

and a number of unit cells $Nd=1/f<A/\sigma_0N$ in each domain. For $\sigma_0=10^{-24}$ cm^2 this number is too small for any macroscopic target ($N_d<10^2$A, $N=10^{22}$); the domains are not well defined, such that f approaches unity and the scattering tends to an incoherent scattering.

We conclude that the quantum-mechanical scattering cannot appear for large single-particle cross sections, like $\sigma_0=10^{-24}cm^2$. We note that for neutrinos ($\sigma_0=10^{-44}cm^2$) $f>10^{-22}/A$, $N_d<10^{22}A$ and we may have one domain in the whole target.

The coherent scattering occurs for neutrinos in a crystal precisely due to the small neutrino cross-section σ_0. The above considerations apply also to a polycrystalline target, where f is limited, in addition, by the size of the crystallites and, consequently, the cross-section is much diminished.

Classical scattering

If in equation (2) is not satisfied (*i.e.*, if $\tau<\sigma t_{eq}$), the incident particles see the target "atoms" (atomic nuclei, molecules, unit cells) as independent scatters. We call this scattering a classical scattering. We can see that this condition implies low energies. The initial wave function is

$$\frac{1}{\sqrt{V}}e^{ikR}\frac{1}{\sqrt{V}}e^{iKR_c}$$

(up to wave functions corresponding to the internal degrees of freedom), where k is the wave vector of the incident particle, K is the wave vector of the center of mass, R_c is the position of the center of mass and $R=r+R_c$. The matrix elements of the interaction given by equation (1),

$$H_{fi}\sim\sum_{i=1}^{N}e^{i(k-k')r_i}\delta_{K'+k',K+k}\ , \qquad (19)$$

includes the form-factor

$$F(k-k')=\sum_{i=1}^{N}e^{i(k-k')r_i}\ , \qquad (20)$$

Where k′ is the wave vector of the scattered particle and K′ is the final wave vector of the center of mass. We can see that the total momentum, including the momentum of the center of mass, is conserved. For a crystal

$$F(k-k')=N\delta_{k',k+g}$$

Where g is a reciprocal vector of the lattice. For an amorphous target g=0. It follows that we have diffraction peaks. For the cross-section of a peak we get $d\sigma_g\sim N^2 do_g$, where the solid angle o_g extends to the range $\Delta_{og}\simeq N^{-2/3}(2/dk')^2$, where d is the mean distance between unit cells (scatterers). It follows $\sigma_g\sim N^{4/3}/(dk')^2$. On the other hand the number of peaks is $\simeq (dk')^2\gg 1$, such that the total cross-section is

$$\sigma=N^{4/3}\sigma_0\ , \qquad (21)$$

where σ_0 is the single-particle cross-section. For one peak σ should be divided by the number of peaks. As it is well known, this cross-section is affected by the Debye-Waller factor and diffuse scattering. According to equation (15) for $nd=N/N_d=fN$ domains the cross-section is

$$\overline{\sigma}=n_d N_d^{4/3}\sigma_0=\frac{\sigma_0 N}{f^{1/3}}\ . \qquad (22)$$

For f=1/N we recover the total cross-section $N^{4/3}\sigma_0$ of one domain, while for f=1 the cross-section reduces to $N\sigma_0$ of an incoherent scattering. The conditions $\bar{\tau}=1/\Phi\bar{\sigma}<$ □teq and $\bar{\sigma}<A$ to

$$\frac{\sigma_0 N}{A}<f^{1/3}<\frac{\hbar\Phi}{\Delta E}\sigma_0 N\ , \qquad (23)$$

which implies $N_d=1/f<(A/\sigma_0 N)^3$ ($\bar{\sigma}<$ A). For $\sigma_0=10^{-24}$ cm^2 and $N=10^{22}$ the number of unit cells $N_d<10^6A^3$ in a domain may indicate well-defined domains for macroscopic targets. The force is bounded from above according to the inequality $F<\Phi Ap$. The in equations (23) imply $\Delta E<\hbar\Phi A$, *i.e.* $\Delta E<10^{-15}$Aerg ($\Phi=10^{12}/cm^2\cdot s$). Therefore, we may set $\Delta E=vp$ and $p<\hbar\Phi/v$ A. Consequently, this upper bound is given by ($v=10^4cm/s$)

$$F<\frac{\hbar}{v}(\Phi A)^2\simeq 10^{-7}A^2 dyn$$

For any reasonably large area A and flux density Φ it is difficult to satisfy these conditions ($p<\hbar\Phi A/v$) and to measure such a force in current experimental situations. For higher energies the energy transfer is higher and we need to apply in equations (18); the large cross-section in this case leads to an incoherent scattering.

Conclusion

A quantum-mechanical scattering is identified in certain conditions in macroscopic targets, which may lead to a coherent cross-section in high-purity, stiff mono crystals. This coherent scattering may explain the Weber-type experiments of neutrino detection by using sapphire mono crystals. The coherent-scattering conditions are not fulfilled by other types of projectiles, with higher single-particle cross-sections (like X, gamma or neutrons). In these cases a classical diffraction may occur in crystals, which generates a weak force, at the limit of detection.

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Statement

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