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Editorial

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An Overview of the Foundations of the Hypergroup Theory

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Abstract

This paper is written within the framework of the Special Issue of Mathematics entitled "Hypercompositional Algebra and Applications", and focuses on the presentation of the essential principles of the hypergroup, which is that the prominent structure of hypercompositional algebra. Within the beginning, it reveals the structural relation between two fundamental entities of abstract algebra, the group and therefore the hypergroup. Next, it presents the several sorts of hypergroups, which derive from the enrichment of the hypergroup with additional axioms besides those it had been initially equipped with, alongside their fundamental properties. Furthermore, it analyzes and studies the varied subhypergroups which will be defined in hypergroups together with their ability to decompose the hypergroups into cosets. The exploration of this far-reaching concept highlights the particularity of the hypergroup theory versus the abstract pure mathematics, and demonstrates the various techniques and special tools that has got to be developed so as to realize results on hypercompositional algebra.

Keywords: Group, Hyper Group, Suphypergroup, Cosets

Introduction

The early years of the 20th century brought the top of determinism and certainty to science. The emergence of quantum physics rocked the well-being of Newtonian mechanics, which was founded by Newton in Philosophiæ Naturalis Principia Mathematica. In 1927, Werner Heisenberg developed his indeterminacy principle while performing on the mathematical foundations of quantum physics. On the opposite hand, in 1931 Kurt Gödel published his two incompleteness theorems, thus giving an end to David Hilbert's mathematical dreams and to the attempts that are culminating with Principia Mathematica of Russell. In 1933, Andrey Kolmogorov published his book Foundations of the idea of Probability, establishing the fashionable axiomatic foundations of applied mathematics. within the same decade the uncertainty invaded algebra also. A young French mathematician, Frédéric Marty (1911–1940), during the 8th Congress of Scandinavian Mathematicians, held in Stockholm in 1934, introduced an algebraic structure during which the rule of synthesizing elements results to a group of elements rather than one element. He called this structure hypergroup. Marty was killed at the age of 29, when his airplane was hit over the Baltic, while he was within the military during war II. His mathematical heritage on hypergroups was only three papers. However, other started performing on hypergroups shortly thereafter. Thus, hypercompositional algebra came into being.

Hypercompositional algebra is that the branch of abstract algebra that deals with structures equipped with multivalued operations. Multivalued operations, also called hyperoperations or hypercompositions, are laws of synthesis of the weather of a nonempty set, which associates a group of elements, rather than one element, to each pair of elements.

The fundamental structure of hypercompositional algebra is that the hypergroup. This paper enlightens the structural relation between the groups and therefore the hypergroups. The study of such relationships is at the guts of structuralism. Structuralism is predicated on the thought that the weather of a system under study aren't important, and only the relationships and structures among them are significant. Because it is proved during this paper, the axioms of groups and hypergroups are an equivalent, while these algebraic entities' difference is predicated on the connection between their elements, which is made by the law of synthesis. In groups, the law of synthesis of any two elements may be a composition, i.e., one element, while in hypergroups it's a hypercomposition, that is, a group of elements.

The next section of this paper generalizes the notion of magma, which was introduced in Éléments de Mathématique, Algèbre by Nicolas Bourbaki, then will include algebraic structures with hypercomposition. The third section presents a unified definition of the group and therefore the hypergroup. This definition of the group isn't included in any pure mathematics book, and its equivalence to the already-known ones is proved within the fourth section. The fifth section presents another, equivalent definition of the hypergroup, while certain of its fundamental properties are proved. As these properties derive directly from the axioms of the hypergroup, they outline the strength of those axioms. So, as an example it's shown that the dominant within the bibliography definition of the hypercomposition includes redundant assumptions. The restriction that a hypercomposition may be a mapping from $E \times E$ into the family of nonempty subsets of E is needless, since, within the hypergroups, the results of the hypercomposition is proved to be always a nonvoid set. The sixth section deals with differing types of hypergroups. The law of synthesis imposes a generality on the hypergroup, which allows its enrichment with more axioms. This creates a mess of special hypergroups with many and interesting properties and applications. The join space is one among them.

It had been introduced by W. Prenowitz so as to review geometry with the tools of hypercompositional algebra, and lots of other researchers adopted this approach. Another one is that the fortified join hypergroup, which was introduced by G. Massouros in his study of the idea of formal languages and automata, and he was followed by other authors who continued during this direction. another is that the canonical hypergroup, which is that the additive a part of the hyperfield that was employed by M. Krasner because the proper algebraic tool so as to define a particular approximation of complete valued fields by sequences of such fields. This hypergroup was utilized in the study of geometry. Moreover, the canonical hypergroup became a part of other hypercompositional structures just like the hypermodule and therefore the vector hyperspace. It's shown that analytic projective geometries and Euclidean spherical geometries are often considered as special hypermodules. Furthermore, the

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hyperfields were connected to the conic sections via variety of papers, where the definition of an elliptic curve over a field F was naturally extended to the definition of an elliptic hypercurve over a quotient Krasner's hyperfield. The conclusions obtained are often applied to cryptography also. Moreover, D. Freni in extended the utilization of the hypergroup in additional general geometric structures, called geometric spaces. Also, hypergroups are utilized in many other research areas and recently, in social sciences and in an algebraization of logical systems. The seventh section refers to subhypergroups. A far-reaching concept of abstract pure mathematics is that the idea of the decomposition of a gaggle into cosets by any of its subgroups. This idea becomes far more complicated within the case of hypergroups. The decomposition of the hypergroups can't be addressed during a similar uniform way as within the groups. So, during this section, and counting on its specific type, the decomposition of a hypergroup to cosets is treated with the utilization of invertible, closed, reflexive, or symmetric subhypergroups.

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