



Training the Brain to do Arithmetic: The Gateway to Becoming a Lifelong Learner of Mathematics

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Abstract

In this paper two models of learning from developmental psychology, implicit and explicit learning, are described. The descriptions focus on developing an understanding of how the brain works in storing and later accessing new information in each of these models. The strengths and weaknesses of each learning paradigm in general, and the specific benefits of each in the learning of mathematics, are discussed. Finally, connections between these two models and traditional and modern school curriculum in mathematics are discussed with recommendations for future teaching and learning practices that blend the two.

Keywords

Automaticity; Cognitive psychology; Explicit and implicit learning; Intervention psychology; Mathematics; Modes of instruction; Remedial mathematics

Introduction

In “A Case for Automatic Recall”, I describe the structure of a highly successful remedial mathematics program offered at Memorial University’s Mathematics Learning Centre [1]. One of the pivotal principles from cognitive psychology mentioned in that paper is the development of automatic recall of basic arithmetic and algebraic skills.

In this paper, I present a model of a general social learning context from developmental psychology that helps elucidate the natural learning orientations of children long before they enter a formal educational setting. These two learning orientations are broadly described as implicit and explicit learning in the literature of developmental psychology [2]. If teachers in the formal educational system, particularly in the early grades, are aware of the differences in these two learning orientations and the strengths and weaknesses of each for different subject topics, then teachers will be better able to assist their students in directing their learning to facilitate more productive lifelong learning trajectories [3,4].

Implicit learning

Implicit learning is based on patterning without any specific

awareness of underlying structural rules or laws. For example, children learn to speak their first language implicitly. They hear adults speaking all around them on a daily basis and simply learn to imitate what they hear. They have no specific knowledge of ‘vocabulary’ as such, or rules of grammar, yet they successfully learn to communicate in their first language long before they enter any formal educational setting.

The strength of this learning pattern is that it is completely natural. A child simply coexists with her/his family and learns to speak. The child is not forced into any regimentation that is strict or unnatural. The child plays, the child eats, the child moves about the community with her/his parents, and as s/he participates in all these natural activities, s/he learns how to speak and communicate in her/his first language.

There are other common examples of activities that are usually learned implicitly. For example, dance movements are most often learned implicitly. An instructor demonstrates a movement and students try to imitate the movement. The instructor repeats the movement and the students adjust what they do to imitate the movement if necessary. This process is repeated until the instructor is satisfied that most of the students are performing a reasonable version of the ideal movement. Then the instructor moves on to a demonstration of the second element of the dance sequence and gives the students the same repeated opportunity to imitate the second element, and so on. This dance sequence is learned completely by imitation and the end result is ideally a well-executed sequence of these particular movements.

A major weakness of this learning pattern is that the specific knowledge of the first learning experience is not *transferable* to any subsequent related learning experience. For example, from a child’s learning experience of his/her first language, the child has no specific knowledge of the grammar of the language. The child has no knowledge of the different categories of vocabulary as nouns, verbs, and prepositions, and no knowledge of the rules that specify the structure for using that vocabulary to construct meaningful sentences. This means that when a child tries to learn a second language based on his/her knowledge of his/her first language, there is nothing to build on – it is a completely new learning experience on a blank language slate [5].

Another weakness of this learning pattern is a lack of retention if the implicitly learned skill is used infrequently. When I first began my study of the differences between implicit and explicit learning, I placed myself in a completely unfamiliar implicit learning environment. I went to Ghana for three weeks to study African drumming and dancing. Since I am more comfortable in explicit learning environments, I often teach myself explicitly by ‘translating’ instruction I experience implicitly into explicit rules. However this was not possible for African rhythms. Unlike any other movement or music I had experienced in my life, the African rhythms were unpredictable. Training in African drumming entailed gathering under the trees in the village for an hour every day with a group of villagers, the master drummer, and an assortment of percussion instruments. We would each take turns using different instruments and we were simply invited to ‘follow along’ with the master drummer, imitating as best as we could

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whatever rhythm we could detect from the cacophony of various percussion instruments being played. Trying to follow along and imitate was difficult enough, but after a few sessions I was traumatized when I was asked in turn to be the rhythm leader. Horror of horrors, I had no idea of what exactly was expected of me, but I drummed away adequately (at least I assumed what I was doing was adequate since all the other participants kept drumming with me), but I certainly was never able to repeat that performance. Furthermore, now when I try to repeat the various rhythms, nothing comes to my mind apart from the beginning strokes on the gankogui which I vaguely recall as “weak, strong, pause, strong, pause, pause, weak, weak (repeat)”. No memory of anything else comes to mind even as I endlessly repeat this beginning rhythm. The master drummer never described the rhythm above for us. He simply demonstrated it repeatedly until I and all the other participants could imitate it. It was me who, in the interests of preserving the knowledge for research purposes, created a sequence of words that I would be able to ‘translate’ into action in my own mind, at least for the beginning rhythm.

Another common example of implicit learning is in athletics. A skating or hockey coach will demonstrate a hockey stop and then invite members of his/her team to imitate his/her movements. The team members try to imitate what they have observed while the coach tries to provide corrections, usually by repeated demonstrations. In time, most skaters will eventually learn how to do a hockey stop, but if the repeated demonstration approaches (i.e.) implicit learning experience, fails, then no other help is available to the learner.

Explicit learning

In sharp contrast to implicit learning, explicit learning is rules oriented. For example, most people learn a second language explicitly by studying vocabulary and rules of grammar. An obvious exception to this is an immersion experience in a second language, for example, in a family setting with bilingual parents who may be speaking in one language at home, and the child experiencing a second language at daycare and later school. Interestingly, because both these language experiences are implicit, this child will be in no better position to learn a third language later on because of the lack of transferability of knowledge from implicit learning experiences. However, once an individual has explicitly learned the structure of one language in terms of specific elements of vocabulary like nouns, verbs and their conjugation, prepositions, and the language’s grammar in general, then that knowledge will definitely be transferable to the learning of yet another language.

In many cases certain skills can be learned either implicitly or explicitly. Individual learners often lean more to one way of learning than the other, but any learner who understands the learning processes behind each can choose to switch between these two modes according to his/her expected use of the knowledge being sought [2].

In the authors describe their innovative approach to teaching Romani students content in molecular chemistry through a first exposure to creating physical representations of molecules using different sizes, shapes and colors of component atoms. Since Romani culture is primarily based upon physical contact with nature and the environment in general, this tactile experience with the physical models in an environment more akin to group play than an unfamiliar and alienating formal sit-down educational setting, appealed to the students’ innate understanding of learning through imitation and experience rather than formal rules governing molecular structure. This implicit learning experience provided the learners with a non-

threatening interest in the subject itself, and provided these astute educators with an enhanced opportunity to lead the Romani students through the transition to an explicit learning experience that entailed developing an awareness of the underlying formal models for the structure of molecules and the manipulation of chemical equations.

Examples of transitions between implicit and explicit learning experiences

Many athletic movements are amenable to explicit learning. For example, at Nick Bollettieri’s Tennis Academy in Florida that I attended as an adult learner, the standard approach to teaching beginners the forehand and backhand strokes is simply to repeatedly demonstrate the movement and invite learners to imitate the instructor’s movement. As the students attempt to imitate the instructor’s movement, the instructor moves about the court making corrections to individual executions of these strokes and the individuals try to adapt their movement following the corrections of the instructor. It is important to note that the success of this approach depends heavily on the acuteness of the student’s attention to details like the major axes of rotation in executing the stroke, that is, wrist, elbow, shoulder, and/or hip. This observation leads us to speculate that a more explicit approach to teaching the forehand and backhand strokes would have the instructor describe the strokes to the learners specifically in terms of the various axes of rotation and an articulation of the most appropriate weight transfer in the feet and legs as the stroke is completed. This more explicit approach to athletic training is definitely used in the advanced training of Olympian athletes, for example. The fine-tuning of athletic performance at this level of competition demands attention to detail that goes far beyond mere observation and imitation.

It is also possible, but perhaps less efficient and less artistic at least initially, to teach and learn many dance movements explicitly. For example, I often teach one element of a flamenco dance routine explicitly in workshops that I deliver on the topic of this paper. I demonstrate the complete movement first, and then I break the movement into two key components, rhythm and foot patterns. In this complex flamenco movement element, I further deconstruct both the rhythm and the foot patterns into simpler components as follows below.

The rhythm has two distinct components. The first component is a strong beat followed by a weak beat. The second component is a strong beat followed by two weak beats. The complete rhythm pattern alternates two three-beat components followed by one two-beat component.

I teach the rhythm patterns first using clapping. Then I begin to teach the foot movements one rhythmic component at a time. There are only three rules to follow for the foot movements. Firstly, the individual foot movements always alternate between the left foot and the right foot. Secondly, each strong beat is a stomp with the whole foot. Lastly, each weak beat is a tap with the heel of the foot while keeping the toe of the same foot planted on the floor.

When we put all the information presented in the previous two paragraphs together, we get the following description starting with the right foot:

Stomp with the right foot, tap the heel of the left foot, tap the heel of the right foot, stomp with the left foot, tap the heel of the right foot, tap the heel of the left foot, stomp with the right foot, tap the heel of the left foot. Repeat.

I always teach each component separately first, and then gradually integrate the components, one by one.

Theoretically, there is not much here that needs to be ‘memorized’. However, training the body to execute ‘only’ the three foot movement rules above concurrently according to the specified rhythm and speed characteristic of this flamenco dance, is quite a different matter. In fact, as I demonstrate in my workshop, this will only come about with extensive repeated articulation of first the component skills and then the aggregated components at very gradually increasing speeds, something that is definitely not attainable in a 40-minute workshop.

This workshop experience also demonstrates another important aspect of the intrinsic relationship between implicit and explicit learning. Even when initial exposure to a skill is explicitly articulated as above, ultimately the successful execution of that knowledge is accessed from memory through automatic recall and is more akin to implicit knowledge than explicit knowledge. The key difference is that the explicit procedural knowledge of the rules that preceded the development of automatic recall can assist the learner in more quickly learning small variations of these rules (like starting with the left foot as opposed to the right), and in avoiding using similar looking rules in inappropriate settings. A good example of the use of a similar looking rule in an inappropriate setting coming from an implicit learning experience in mathematics is given in the next paragraph.

In beginning algebra, students learn the laws of exponents. One of the four foundational laws is $(xy)^2 = x^2y^2$. A common error that occurs in the use of this law is when it is inappropriately applied to the operation of addition or subtraction within the bracket (e.g.) in simplifying $(x + y)^2$ as $x^2 + y^2$. This most often occurs when a learner has ‘rote-memorized’ the laws of exponents, rote-memorization being the characteristic mode of retention following an implicit learning experience. If, however, a learner is first exposed to the conceptual models underlying these laws of exponents and hence understands that exponentiation is repeated *multiplication*, then s/he is less likely to use the ‘automatized’ rule, $(xy)^2$, when the operation within the bracket is addition or subtraction and not multiplication.

Rote memorization versus automatic recall in mathematics

The ongoing debate in education about the role of ‘memorization’ in the learning of mathematics has been misguided in two fundamental ways:

1. No distinction has been made between ‘rote memorization’ and ‘automatic recall’.
2. In terms of the applicable model from developmental psychology that is at play here, the arguments made to justify or refute ‘memorization’ should properly be made in the context of implicit versus explicit learning.

Advanced mathematics like algebra is not well suited to implicit learning by its very foundational principles. While mathematics is definitely observable informally in nature, its primary purpose in the development of advanced thought in our modern world lies in using the very formalized structure of logic to model both observed and hypothesized analytical systems on a microscopic or a macroscopic level. The mathematician studies different formal systems for numbers by identifying what key aspects of number systems are pivotal in differentiating these systems. For example, the whole number system (e.g.) the counting numbers, 1, 2, 3, 4, 5, etc., is fundamentally different from the integer number system

(e.g.) 0, +1, +2, +3, etc, for the following reason. If you subtract bigger numbers from smaller numbers in the whole number system, you do not get an answer in that system ($3-5 = -2$ is not a counting number), but no matter what two numbers you subtract in the integer number system, you get another integer, that is, another number in that system. In the study of mathematics, this is of monumental importance. Subtraction is a basic arithmetic operation, and within any formal logical system, you cannot perform a meaningful analysis of phenomenon when you cannot be sure that you are staying within that system. This particular difference between the whole number system and the integer number system can be problematic for students who have been given an implicit learning experience of numbers through discovery-based learning. If the learner’s experience of ‘subtraction’ is never taken beyond that of ‘take-away’ to the more abstract ‘adding the additive inverse’ of the integer number system, then that learner is likely to have difficulty dealing with (i) leading negatives in algebraic expressions; (ii) negative exponents; (iii) the laws of integer multiplication and division; (iv) the distributive law of multiplication with respect to addition/subtraction; and so on. From a motivational perspective, it is important that learners are exposed to the relationship between formal mathematical systems and natural phenomena in their environment, but from the perspective of the lifelong learning trajectory in mathematics of the student, a shift to the explicit learning experience is imperative.

Conclusion

In summary, IMPLICIT LEARNING leads to ROTE MEMORIZATION, no transference, and poor retention for skills used infrequently. In contrast, EXPLICIT LEARNING can lead to AUTOMATIC RECALL with well-structured practice, transference of the skill to a variety of contexts, and good retention in general. However, given that very young children are brought up in a natural social environment where learning is implicit, it is crucial that teachers are aware of the effect that this can have on the learning that children experience in a formal educational setting. Especially in mathematics where implicit learning will not provide adequate scaffolding for the required learning progression to abstract mathematical models, teachers must be prepared to gradually reorient the learner’s trajectory from implicit to explicit learning, especially in the early grades. This will then provide the learner with the best of both worlds, that is, the deep understanding that comes from knowing the intrinsic connections of mathematics to the natural world, and the power of the expansive applicability that comes from a familiarity with the abstract mathematical models.

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