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# A 2d Fea for Aquifer Boundary Inverse Problem: Co-Simulating Groundwater Flow in Lebna Watershed-Tunisia Using Comsol-Matlab

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## Abstract

This study was conducted in the Lebna side of Plio-quaternary aquifer belonging to Cape Bon in the Northeastern side of Tunisia. This coastal aquifer is often inaccessible and does not have enough information about its parameters such as hydraulic head and hydraulic fluxes.

This paper aims to treat the Partial Differential Equations (PDE) of groundwater flow problem. Variational method based Finite Elements (FE) has been used to solve the Cauchy problem or inverse problem. This problem accommodate an inverse algorithm developed by Andrieux et al. in order to identify the missing data as hydraulic head and hydraulic fluxes on a part of an aquifer boundaries where data are non-available.

Cauchy problem, which is known as an ill-posed problem, has been divided into two well-posed sub-problems. Then the role of the inverse algorithm is shortly solutions optimization; it works on solving those sub problems; each one is supposed to find out specific data, one for hydraulic head and the other for hydraulic fluxes. Only the nearly equivalent to exact data have been considered as optimal solutions, which are determined based on minimizing an energy like-error in each simulation.

In this case of study, a forward problem of Darcy Flow has been solved using measurement data as input through Comsol Multiphysics, a FE based modeling software. The outputs of the forward simulation have been utilized as the entry data written within the inverse algorithm in Matlab for the aim of simulating data where requested.

In the general context of aquifer, modeling the data completion problem is very helpful to understand the boundary conditions, which are determined in the resolution.

Authors and others have studied this kind of problem in an academic framework but to our knowledge, it is the first work with a real case of aquifer

Computed results have been compared to measurement data, exact ones and similarity has been noticed which could prove the efficiency of the method used in this context since 2006. Thus, RMSE is about 0.055 and a percent error varies between 0 and 14 % for hydraulic head.

Keywords: Inverse Problem; Data Completion; Darcy Flow; Forward Model; Minimization; Finite Element Method; Lebna Aquifer; Tunisia

## Introduction

It is widely recognized that water shortage has been noticed as an important issue in the world. In Tunisia, quantity deterioration has been discovered in the coastal aquifer of Cap Bon since the 60s.

Overexploitation of groundwater resources nationally has exceeded 120% on average. Therefore, it is important to manage these resources for the future of Tunisia. Indeed, hidden phenomena of underground flow are preceded in aquifer neither directly nor continuously.

Over the last thirty years, mathematical modeling techniques have been intensively used in hydrogeology for management studies, exploitation and remediation studies of aquifers, seawater intrusion, determination of protection zones around catchment wells and other groundwater issues [1]. In addition, analytical methods have provided precise solutions to the problem expressed explicitly using known mathematical functions. However, researchers realized limitation of the analytical resolution of complex hydrogeological problems by assumptions that were often too simplistic. Many of them started to use digital inverse methods (De Marsily, Carrera and Neuman). These methods, though requiring larger computational efforts, provide us with powerful tools for analysis and simulation that solve underground flow and contaminant transport problems in 2D or 3D systems strongly heterogeneous, isotropic or non-isotropic and with complex geometry.

This is the case of the Cap Bon aquifer, which was the subject of several studies with different objectives, aspects and approaches. Located in the Northeast of Cape Bon, the study area belongs to the Eastern Coast of Cap Bon, part of the Plio-Quaternary water table in the Lebna plain. This aquifer is often inaccessible and does not have enough information on its state; such as hydraulic head and fluxes. In this study, to solve the problem of lack of data we resorted to data completion method, which requires particular techniques mathematical resolution [2].

The aim of this study is to provide a numerical model for predicting groundwater flow as well as to solve an inverse problem of data completion in order to identify some missing boundary conditions. To validate this model, both inverse and forward results have been compared to the findings of Ennabli in 1970.

### Study area

The study area is located in the eastern part of the city of Nabeul; it is bounded by the Mediterranean Sea from the eastern side, Sidi Abderrahmen Mountain from the Western side, Chiba River from the Southwestern side and Kelibia prefecture from the Northeastern [3]. In addition, Lebna River is crossing through it. It has an area of 203

Km2. The climate in the study area is mostly upper semi-arid and sub wet. Annual rainfall is equal to 450 mm, temperature is around 22 °C and wind speed is ranging from 18 to 24 Km/h.

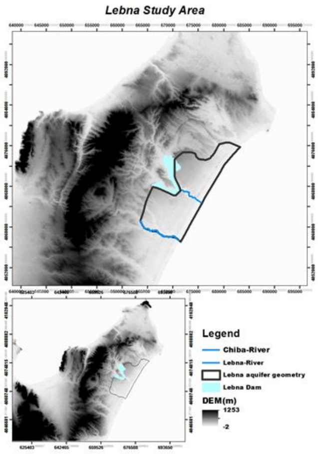


**Figure 1:** Study area

### Inverse modeling background

Considering a model; a PDE of natural or artificial process (groundwater flow, solute transport, heat transpor) with a set data; known coefficients and few known measurement points (observations).

Inverse modeling is an application of this data as input to estimate the same kind of data on other observation nearby points where data is unavailable, which in this context is called the forward model. This model estimates missing data but with a negligible accuracy. Therefore, we use the obtained solutions as an input in order to build another model called inverse model. This model is obtained with respect to entry data of forward model which correspond a data of reference while consider the forward outputs as a basis data. Thus, combining data from both modeling is the purpose in order to reach an optimal solution with higher accuracy. Therefore, Cauchy problem has been considered a more realistic presentation of the PDE governing groundwater. The called problem, qualified as an ill-posed problem, is introduced for the aim of data completion [4].

The aspect of well-posed problem introduced by Hadamard, this kind of problem should satisfy the stability and unicity of solution which is impossible to exist. Thus, subdividing such problem has been considered a useful tool where sub problems could satisfy those conditions for their proper solution.

During the last years, variational techniques, which derived from Tikhonov's regularization methods, have been considered the most attractive one regarding the accuracy that provides. Therefore, this paper has adapted the approach of Andrieux, one of the reliable technics in the context of data completion [5].

It consists on dividing the Cauchy problem in which the couple of data, hydraulic head and hydraulic fluxes, are unable to be found in the same time with a high certitude [6]. The ill-posed problem was replaced by two well-posed problems where one gives head values as solutions, called Neumann Problem (PN), and the other one the fluxes as solutions, called Dirichlet problem because it imposes the head values as a condition in the boundary where data are unavailable.

## Methodology

Since we have two problems to be solved, we started by solving the forward problem (PF). Simply we just considered the measurment data as the input for the PDE based model in Comsol. In the second part, Cauchy problem has been solved using, the output of (PF) simulated on Comsol [7].

### Forward Simulation

Forward simulation is about solving the strong from PDE of Darcy flow. Two boundary conditions, important for groundwater modeling, have been examined: Dirichlet and Neumann through two-dimensional, steady simulations.

The following Darcy Flow problem has been considered as the forward problem (PF):

$$\begin{cases} \nabla.(-T\nabla h) = f & \text{in } \Omega \\ h = H & \text{on } \Gamma_D \,;\, \Gamma_D \cup \Gamma_N = \partial\Omega \\ -T\nabla_n h = \Phi & \text{on } \Gamma_N \end{cases}$$

Where Ω: The aquifer domain

ΓD: The aquifer boundary with Dirichlet boundary conditions.

ΓN: The aquifer boundary with Neumann boundary conditions.

∂Ω: The aquifer boundary, which is the union of two boundary ΓN and ΓD.

In order to solve the problem (PF) we proceed by Comsol Multiphysics a based FEA software. Thus, the geometry of the aquifer has been imported as Data Exchange file then divided into subdomain depending on the geological parameters, the transmissivity values [8]. In addition, at each subdomain, we introduced a pumping rate (the sum of the pumped volume of all the wells belong to a subdomain). Then, we extracted the following boundary conditions from the Piezometric and geological maps established by Ennabli at each point where data has been defined as following:

Dirichlet Condition along the shoreline: a constant Hydraulic head equal to 0m on the Mediterranean boundary (the east side of the considered aquifer);

Dirichlet Condition on the west side (Mountain of Sidi Abderrahmen): h is considered as time-dependent variable waving between 0 and 50 m;

Neumann Condition on the North-West side: for impervious side, we consider fluxes;

Neumann Condition for Rivers: hydraulic fluxes are maintained constant and equal to 2 m3/s and 4m3/s for Chiba River and Lebna one respectively.

After all, meshing has been applied to the domain in order to subdivide it into variably volume rectangular finite elements where the PDE has been solved in each node with respect to nodes nearby (degrees of freedom) then getting the problem (PF) solved and presenting results or even editing it, this is the post processing [9].

### Inverse Simulation

Once we have obtained the outputs from the forward simulation, we proceed to consider them as the entry data for the inverse simulation. Based on this, we solve P in order to handle the issue of lack of data in the inaccessible boundary of the aquifer; the Dam of Lebna (Figure 1 and Figure 2). Thus, we properly solve a problem of data completion.

The problem (P) is written as following:

$$(P)\begin{cases} -\nabla.(T\nabla h) = f \text{ in } \Omega \\ h = H \quad \text{on } \Gamma_D \\ T\nabla_n h = \phi \quad \text{on } \Gamma_N \\ h = \overline{H} \quad \text{on } \Gamma_m \\ T\nabla_n h = \overline{\phi} \quad \text{on } \Gamma_m \\ h = ? \quad \text{on } \Gamma_u \\ T\nabla_n h = ? \quad \text{on } \Gamma_u \end{cases} \qquad \Gamma_D \cup \Gamma_N \cup \Gamma_m \cup \Gamma_u = \partial\Omega$$

Where Γu : Boudary with unknown data

ΓD : Dirichlet Boundary

ΓN : Neumann Boundary

Γm : Boudary with measured data

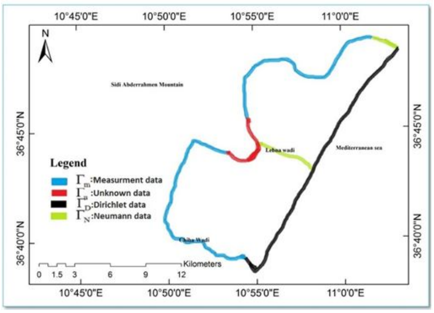


**Figure 2:** aquifer boundaries corresponding to Cauchy problem Boundary conditions.

The consistency of FEM gave a focus on dividing the Cauchy problem P into two-mixed problems (sub problems) where each one is supposed to find out solution relative to each boundary condition data;

Dirichlet problem (PD) related to Dirichlet conditions known on the boundary of data measurement Γm: (PD) is supposed to give Neumann data as solution; hydraulic head

Neumann problem (PN) related to Neumann conditions known on Γm: (PN) is supposed to give Dirichlet data as solution; hydraulic fluxes.

$$(P_N)\begin{cases} -\nabla.(T\nabla h_N) = f \text{ in } \Omega \\ h_N = H \quad \text{on } \Gamma_D \\ T\nabla_n h_N = \phi \quad \text{on } \Gamma_N \\ T\nabla_n h = \overline{\phi} \quad \text{on } \Gamma_m \\ h_N = \tau \quad \text{on } \Gamma_u \end{cases} ;$$

$$(P_D)\begin{cases} -\nabla.(T\nabla h_D) = f \text{ in } \Omega \\ h_D = H \quad \text{on } \Gamma_D \\ T\nabla_n h_D = \phi \quad \text{on } \Gamma_N \\ h_D = \overline{H} \quad \text{on } \Gamma_m \\ T\nabla_n h_D = \eta \quad \text{on } \Gamma_u \end{cases} ;$$

$$J(h_D, h_N, f) = \frac{1}{2}\int_{\Omega} T(x,y).(\nabla h_D - \nabla h_N)^2 d\Omega$$

In order to apply a finite element resolution for the 3 problems (PD), (PN) and (P), they have been reformulated into vibrational form where integral partition have been applied on the PDE governing the domain Ω, with respect to the data available on the boundary ∂Ω [10]. Thus, a scripted writing for this reformulation could make a sense and give us the asked solutions. For this reason, Matlab was a useful tool for building a FE simulation for P. An iterative algorithm has been written as following:

Initiate the hydraulic head and fluxes on the aquifer boundary

Solving the strong from of PDE on boundary

Solving the Sub PDE; PD and PN and calculate J from hD and hN

Repeat Sub PDE resolution until reaching convergence (hD and hN converge into h exact)

## Results and Discussions

### Forward model

The forward model (Figure 3) shows the spatial distribution of hydraulic head in the aquifer. Along the shoreline, the model has appeared in dark blue, which shows low values of hydraulic head. On the other hand, high hydraulic head are located in the North-West side bounded by the mountain Sidi Abderrahmen. Otherwise, the flow paths are clearly oriented towards the shoreline.

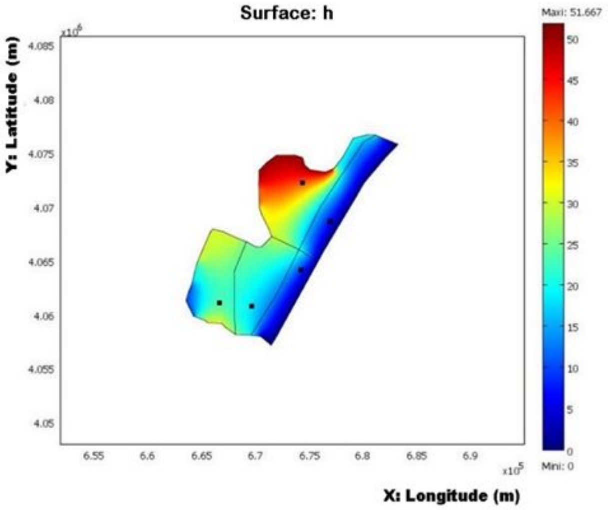


**Figure 3:** Groundwater flow FE-based model.

The figure 4 shows a spatial distribution of hydraulic head. This distribution has been mapped and interpolated using kriging tools in ArcGIS, a deterministic method which aims to determine estimated data in a nearby point.

Thus, hydraulic head varies upward along the aquifer from downstream to upstream, it reached the maximum 54m on the West side, mountain.

If we compare this model to the one computed using Comsol, we could notice similarity. As the same measured data has been used as input while difference only on the way we mapped information. Especially on the boundary, the variation of hydraulic head appears the same way in the two model mapped. Therefore, we could notice that the finite element method shows an efficient tool to estimate information about groundwater flow.

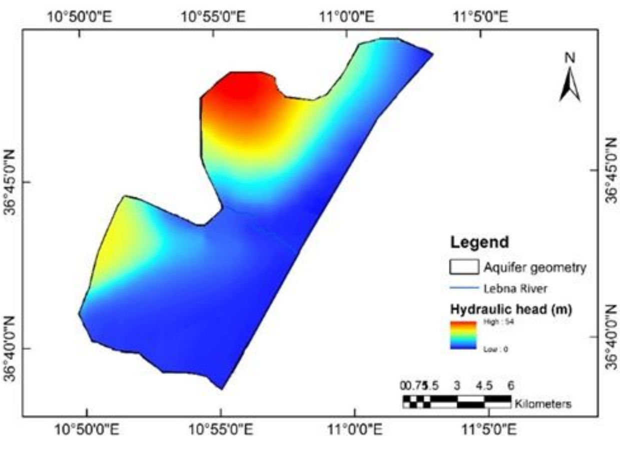


**Figure 4:** Piezometric map Established by Ennabli 1970.

## Inverse model

The inverse model, in the following figure, shows a similarity with the forward model both in the steady state. These results prove the efficiency of the adopted method.

This model shows the variation of the hydraulic head and fluxes respectively on the unknown boundary Γu. The lag between the measured hydraulic head and the computed one seems to be high in the beginning, which might be explained by the transition zone from two kind of boundaries; known data Γm and unknown data Γu respectively for each one. Then the lag comes to disappear which correspond to the convergence forward the exact hydraulic head when reaching the other side of the Γm.

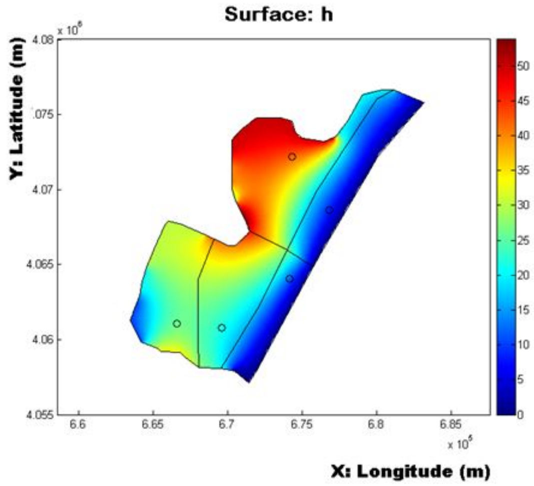


**Figure 5:** Inverse model of groundwater flow in Lebna aquifer.

The following figures show the difference between exact solutions obtained from forward problem and computed ones obtained from Dirichlet and Neumann problem solved on Matlab for both hydraulic head and fluxes data. We could notice that both hydraulic head and fluxes had varied spatially along the boundary Γu as a normal behavior. However, they converge into the exact solutions at the terminal segment of the boundary.

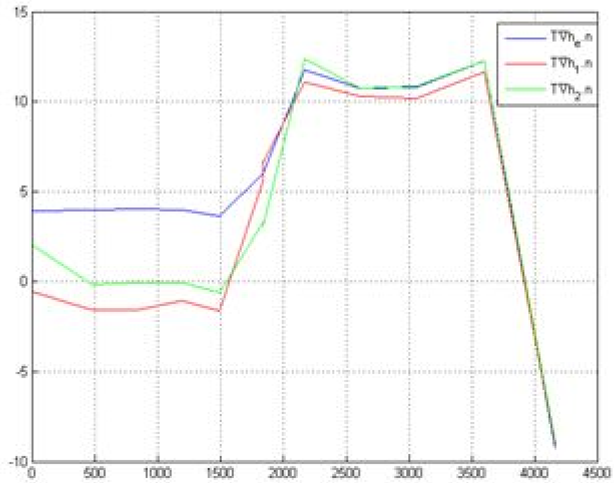

**Figure 6:** (a) Hydraulic fluxes on Γu.

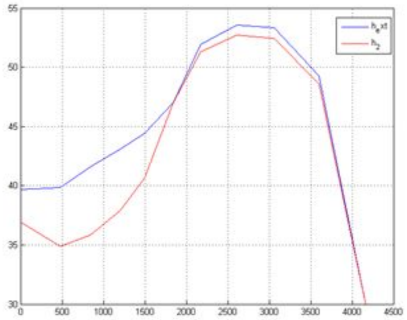

**Figure 6:** (b)Hydraulic head on Γu.

While, the table below shows the variation of error percentage of hydraulic head along the Γu boundary. Error varies between 0 and 13.805%. Minimum errors have been noticed at the end of Γu segment nearby the Γm, the boundary of measurement data.

This might be explained by the convergence tool, which aims to find out solutions equivalent to the exact ones. Thus, the possibility to obtain exact solutions remains stronger as long as we move into the boundary of known data.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| hexact (m) | 36.943 | 34.874 | 35.837 | 37.861 | 40.616 | 52.701 | 48.576 | 51.32 | 52.438 | 47.039 | 30 |
| Hcomputed (m) | 39.699 | 39.862 | 41.575 | 43.043 | 44.426 | 53.565 | 49.29 | 51.91 | 53.334 | 47.039 | 30 |
| Error (%) | 6.943 | 12.514 | 13.803 | 12.039 | 8.577 | 1.612 | 1.45 | 1.136 | 1.679 | 0 | 0 |

**Table 1:** Error percentages of computed hydraulic head

## Conclusion

This research proved the consistency of the Finite Element simulation in a real aquifer case, Lebna aquifer. We assessed groundwater flow using this method. Inverse simulation was a useful tool to handle the issue of lack of data within the aquifer. This method could be a useful tool to solve the problem of the lack of data for decision maker to assess water resources. However, it needs a high accuracy for the data used initially.

## Reference

1. Andrieux S, Baranger TN, Abda BA (2006) Solving cauchy problems by minimizing an energy-like functional. Inverse Probl 22: 115-133.
2. Carrera J, Neuman SP (1986) Estimation of aquifer parameters under transient and steady state conditions: 1. Maximum Likelihood Method Incorporating Prior Information. Water Resour. Res. 22: 199-210.
3. https://www.pseau.org/outils/ouvrages/crs_exploitation_des_eaux_souterraines_2009.pdf
4. Marsily DG (1999) Regards sur 40 ans de problemes inverses en hydrogeologie. Earth Planet Sci 329: 73-87.
5. Hadamard J (1932) Le probleme de Cauchy et les equations aux derivees partielles lineaires hyperbolique. Annali di Matematica 39: 305-334.
6. Sun NZ (1989) Inverse Problem in Groundwater Modeling. (1stedn), Springer Dordrecht.
7. Tarantola A (1987) Inverse problem theory. (1stedn), Methods for Data Fitting and Model Parameter Estimation, Elsevier.
8. https://biblio.ugent.be/publication/139921.
9. Tikhonov AN. Arsenin VY (1977) Solutions to ill posed problems. (1stedn), New York: Winston Wiley.
10. Ziadi A, Hariga NT, Tarhouni J (2019) Mineralization and pollution sources in the coastal aquifer of Lebna, Cap Bon, Tunisia. J African Earth Sci 151: 391-402.