Research and Reports on Mathematics

Research Article

A SCITECHNOL JOURNAL

Generalized Hybrid Block Method for Solving Second Order Ordinary Differential Equations Directly

Kuboye JO[1]*, Omar Z[2], Abolarin OE[1] and Abdelrahim R[3]

Abstract

In this article, three steps block method with three generalised hybrid points is developed for the solution of second order initial value problems of ordinary differential equations. The derivation of this method is achieved through interpolation and collocation technique where power series approximate solution is employed as the basis function. In comparing the new method with existing methods, specific hybrid points are selected for solving some second order initial value problems. The results generated outperform the existing methods in terms of error.

Keywords

Power series; Interpolation; Collocation; Hybrid points; Block method; Second order ordinary differential equations

Introduction

In this paper, the numerical technique for solving directly second order initial value problems of ordinary differential equations (ODEs) of the form

$$y''(x)=f(x, y(x), y'(x)),\ y(x0)=y0,\ y'(x0)=y_1 \qquad (1)$$

is examined. The reduction of (1) to a system of first order ODEs is associated with the following setbacks which include lot of human efforts and many functions to be evaluated per iteration which may jeopardize the accuracy of the method as discussed in Lambert, Fatunla, Brugnano and Trigiante, Awoyemi and Jator [1-5]. The introduction of specific hybrid points between grid points has been considered in the development of numerical methods for solving ODEs directly by some scholars namely Kayode and Adeyeye, Odejide and Adeniran, Sagir, Yap, Ismail and Senu amongst others [6-9].

In order to bring improvement on numerical methods, this article discusses the derivation of a three–step block method with three generalised hybrid points for the solution of (1) directly.

Also in examining the accuracy of the new method and for the purpose of comparison with some existing methods, a specific hybrid point is selected.

***Corresponding author:** Kuboye JO, Department of Mathematics, Faculty of Sciences, Federal University Oye-Ekiti, Ekiti-State, Nigeria, Tel: (234) 8165396888; E-mail: kubbysholly2007@gmail.com

Received: August 18, 2017 **Accepted:** February 23, 2018 **Published:** March 03, 2018

Derivation of the Method

In developing this method, power series of the form

$$y(x)=\sum_{j=0}^{k+5} a_j x^j \qquad (2)$$

Is considered as an approximate solution to Eq. (1), where k=3. Eq. (3) is derived by differentiating Eq. (2) twice to give

$$y''(x)=\sum_{j=2}^{k+5} j(j-1)a_j x^{j-2}=f(x,y,y') \qquad (3)$$

Interpolating Eq.(2) at $x=x=_{n+i}$, i=0,1 and collocating (3) at $x=x=_{n+m}$, m=0,v,1, (v+1),2, (v+2), 3. where 0<v<1. These equations are then combined to give a nonlinear system of equations of the form

$$AX=B \qquad (4)$$

Where

$$A=\begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 & x_n^7 & x_n^8 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 & x_{n+1}^7 & x_{n+1}^8 \\ 0 & 0 & 2 & 6x_n & 12x_n^2 & 20x_n^3 & 30x_n^4 & 45x_n^5 & 56x_n^6 \\ 0 & 0 & 2 & 6x_{n+v} & 12x_{n+v}^2 & 20x_{n+v}^3 & 30x_{n+v}^4 & 45x_{n+v}^5 & 56x_{n+v}^6 \\ 0 & 0 & 2 & 6x_{n+1} & 12x_{n+1}^2 & 20x_{n+1}^3 & 30x_{n+1}^4 & 45x_{n+1}^5 & 56x_{n+1}^6 \\ 0 & 0 & 2 & 6x_{n+(v+1)} & 12x_{n+(v+1)}^2 & 20x_{n+(v+1)}^3 & 30x_{n+(v+1)}^4 & 45x_{n+(v+1)}^5 & 56x_{n+(v+1)}^6 \\ 0 & 0 & 2 & 6x_{n+2} & 12x_{n+2}^2 & 20x_{n+2}^3 & 30x_{n+2}^4 & 45x_{n+2}^5 & 56x_{n+2}^6 \\ 0 & 0 & 2 & 6x_{n+(v+2)} & 12x_{n+(v+2)}^2 & 20x_{n+(v+2)}^3 & 30x_{n+(v+2)}^4 & 45x_{n+(v+2)}^5 & 56x_{n+(v+2)}^6 \\ 0 & 0 & 2 & 6x_{n+3} & 12x_{n+3}^2 & 20x_{n+3}^3 & 30x_{n+3}^4 & 45x_{n+3}^5 & 56x_{n+3}^6 \end{bmatrix}$$

X= $[a, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8]^T$, B= $[y_n, y_{n+1}, f_n, f_{n+v}, f_{n+1}, f_{n+(v+1)}, f_{n+2}, f_{n+(v+2)}, f_{n+3}]^T$

Gaussian elimination technique is used in finding the values of aj's in (4) which are then substituted into (2) to produce a continuous implicit scheme of the form

$$y(t)=\sum_{i=0}^{k-2}\alpha_i(t)y_{n+i}+h^2\left[\sum_{i=0}^{k-1}\eta_{v+i}(t)f_{n+(v+i)}+\sum_{i=0}^{k}\beta_i(t)f_{n+i}\right] \qquad (5)$$

$$t=\frac{x-x_{n+k-1}}{h}$$

$\alpha_0(t)=(-t-1)$

$\alpha_1(t)=(t+2)$

$$\beta_0(t)=\frac{(t+2)(t+1)}{5040v(v+2)(v+1)}\begin{bmatrix}15t^6-60t^5v+15t^5+84t^4v^2+12t^4v-47t^4-42t^3v^3-126t^3v^2 \\ +126t^3v-15t^3+126t^2v^3+18t^2v-t^2-154tv^3-168tv^2-26tv \\ +33t+210v^3+504v^2+42v-97\end{bmatrix}$$

$$\eta_v(t)=\frac{(t+2)(t+1)}{1680v(v-3)(v-1)(v-2)}\begin{bmatrix}15t^6-40t^5v+15t^5+28t^4v^2-20t^4v-47t+140t^3v-15t^3 \\ +126t^2v^2+30t^2v-t^2+98tv^2+90tv+33t-42v^2-210v \\ -97\end{bmatrix}$$

$$\beta_1(t)=\frac{-(t+2)(t+1)}{1680v(v-1)(v+1)}\begin{bmatrix}15t^6-60t^5v+35t^5+84t^4v^2-72t^4v-51t^4-42t^3v^3+252t^3v \\ -85t^3+56t^2v^3-378t^2v^2+88t^2v+77t^2+196tv^3+294tv^2-208tv \\ -61t-700v^3-126v^2+448v+29\end{bmatrix}$$

$$\eta_{v+1}(t)=\frac{-(t+2)(t+1)}{840v(v-1)(v-2)(v+1)}\begin{bmatrix}15t^6-40t^5v+35t^5+28t^4v^2-48t^4v-51t^4+140t^3v-85t^3 \\ -126t^2v^2+96t^2v+77t^2+98t^2v+77t^2+98t^2v^2-8tv-61t-42v^2-168v \\ +29\end{bmatrix}$$

$$\beta_2(t)=\frac{(t+2)(t+1)}{1680v(v-1)(v-2)}\begin{bmatrix}15t^6-60t^5v+55t^5+84t^4v^2-156t^4v+t^4-42t^3v^3+126t^3v \\ -155t^3-14t^2v^3-336t^2v^2+494t^2v-97t^2+266tv^3-504tv^2+226tv \\ +41t+70v^3-336v^2+14v+71\end{bmatrix}$$

$$\eta_{v+2}(t)=\frac{(t+2)(t+1)}{840v(v-1)(v+2)(v+1)}\begin{bmatrix}15t^6-40t^5v+55t^5+28t^4v^2-76t^4v+t^4+140t^3v-155t^3\\-126t^2v^2+222t^2v-97t^2+98t^2v+77t^2+98t^2v^2-106tv+41t-42v^2-126v\\+71\end{bmatrix}$$

$$\beta_3(t)=\frac{-(t+2)(t+1)}{5040v(v-1)(v-2)(v-3)}\begin{bmatrix}15t^6-60t^5v+75t^5+84t^4v^2-240t^4v+109t^4-42t^3v^3+252t^3v^2\\-252t^3v+27t^3-84t^2v^3+126t^2v^2-24t^2v-19t^2+56tv^3-42tv^2\\+16tv+3t+70v^3-126v^2+29\end{bmatrix}$$

In deriving the discrete schemes, Eq. (5) is evaluated at the non-interpolating points, i.e. at $x=x_{n+m}$, $m=v$, $(v+1)$,2, $(v+2)$,3 while the derivative of the scheme is derived by evaluating the derivative of (5) at all the grid points, i.e. at $x=x_{n+m}$, $m=0,v,1$ $(v+1)$,2, $(v+2)$,3. This discrete scheme and its derivative at point x_n are combined in a matrix form

$$Uy_{N+1}=U^0y_N+hU^1y'_{N-1}+h^2[W^0f_N+W^1f_{N+1}] \tag{6}$$

Where

$$y_{N+1}=[y_{n+v},\,y_{n+1}\cdots y_{n+k}]^T,y_{N-1}=[y_{n+(v+k-1)},y_{n+(k-1)},\cdots y_n]^T,y'_{N-1}=[y'_{n+(v+k-1)},y'_{n+(k-1)}\cdots y'_n]^T,$$

$f_N=[f_{n+(v+k-1)},f_{n+(k-1)}\cdots f_n]^T,f_{N+1}=[f_{n+v},f_{n+1},\cdots f_{n+k}]^T$ and U, U^0, U^1, W, W^1 are nxn matrices. Therefore, the inverse of U is multiplied by (6) and this yield the block method (7)

$$y_{n+v}=-E_0y_{n+1}+F_0y_n+hvy'_n+h^2\begin{bmatrix}G_0f_n+H_0f_n+v+I_0f_{n+1}+J_0f_{n+v+1}+K_0f_{n+2}\\+L_0f_{n+v+2}+M_0f_{n+3}\end{bmatrix}$$

$$E_1y_{n+1}=F_1y_n+hy'_n+h^2\begin{bmatrix}G_1f_n+H_1f_{n+v}+I_1f_{n+1}+J_1f_{n+v+1}+K_1f_{n+2}\\+L_1f_{n+v+2}+M_1f_{n+3}\end{bmatrix}$$

$$y_{n+v+1}=-E_2y_{n+1}+F_2y_n+h(v+1)y'_n+h^2\begin{bmatrix}G_2f_n+H_2f_{n+v}+v+I_2f_{n+1}+J_2f_{n+v+1}+K_2f_{n+2}\\+L_2f_{n+v+2}+M_2f_{n+3}\end{bmatrix}$$

$$y_{n+2}=F_3y_n+2hy'_n+h^2\begin{bmatrix}G_3f_n+H_3f_{n+v}+I_3f_{n+1}+J_3f_{n+v+1}+K_3f_{n+2}\\+L_3f_{n+v+2}+M_3f_{n+3}\end{bmatrix} \tag{7}$$

$$y_{n+v+2}=-E_3y_{n+1}+F_4y_n+h(v+2)y'_n+h^2\begin{bmatrix}G_4f_n+H_4f_{n+v}+I_4f_{n+1}+J_4f_{n+v+1}+K_4f_{n+2}\\+L_4f_{n+v+2}+M_4f_{n+3}\end{bmatrix}$$

$$y_{n+3}=-E_4y_{n+1}+F_5y_n+3hy'_n+h^2\begin{bmatrix}G_5f_n+H_5f_{n+v}+I_5f_{n+1}+J_5f_{n+v+1}+K_5f_{n+2}\\+L_5f_{n+v+2}+M_5f_{n+3}\end{bmatrix}$$

Where

$$E_0=\frac{-(5040v^5-10080v^4-35280v^3+40320v^2+60480v)}{(5040v-10080)(v+1)(v+2)(v-3)}-\frac{(-5040v^6+15120v^5+25200v^4-75600v^3-20160v^2+60480v)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$F_0=\frac{-(5040v^5-15120v^4-25200v^3+75600v^2+20160v-60480)}{(5040v-10080)(v+1)(v+2)(v-3)}-\frac{(-5040v^6+15120v^5+25200v^4-75600v^3-20160v^2+60480v)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$G_0=\frac{(-3v^9+45v^8-252v^7+558v^6+588v^5-4494v^4+3182v^3+7677v^8-9215v+1914)}{(5040v-10080)(v+1)(v+2)(v-3)}-\frac{(1358v^6+4956v^5+2944v^4\text{-}19025v^3+3268v^2+11129v-1914v-1914)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$H_0=\frac{-(-9v^8+216v^6+162v^5-1938v^4-1308v^3+7476v^2+8805v+1914)}{(5040v-10080)(v+1)(v+2)(v-3)}-\frac{(882v^4+4656v^3+8751v^2+6891v+1914)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$I_0=\frac{(1596v^6-2142v^5-14226v^4+7791v^3+32835v^2+3108v-5868)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}-\frac{(-9v^9+85v^8-36v^7-1962v^6+2022v^5+13656v^4\text{-}7626v^3-30075v^2+2760v+5868)}{(5040v-10080)(v+1)(v+2)(v-3)}$$

$$J_0=\frac{(-18v^8+120v^7+192v^6-2796v^5+3924v^4+10224v^3-20508v^2-6690v+5868)}{(5040v-10080)(v+1)(v+2)(v-3)}+\frac{(1764v^4+492v^3-13818v^2-12558v+5868)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$K_0=\frac{(-9v^9-33v^8-264v^7+438v^6+2580v^5-654v^4\text{-}8562v^3-5619v^2+1395v+1026)}{(5040v-10080)(v+1)(v+2)(v-3)}-\frac{(546v^6+1008v^5-3684v^4-10503v^3-7014v^2+369v+1026)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$L_0=\frac{-(-9v^8+120v^7-564v^6+822v^5+1662v^4\text{-}6528v^3+5196v^2+1305v-1026)}{(5040v-10080)(v+1)(v+2)(v-3)}-\frac{(882v^4-4164v^3+3891v^2+2331v-1026)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$M_0=\frac{(3v^9+6v^8-48v^7-120v^6+234v^5+696v^4+370v^3-111v^2+70v)}{(5040v-10080)(v+1)(v+2)(v-3)}+\frac{(112v^6+546v^5+886v^4+481v^3-41v^2-70v)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$E_1=\frac{-(5040v^6+15120v^5+25200v^4-75600v^3-20160v^2+60480v)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$F_1=\frac{-(5040v^5+15120v^4+25200v^3-75600v^2-20160v+60480)}{5040v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$G_1=\frac{-(-1358v^6+4956v^5+2944v^4-19025v^8+3268v^2+11129v\text{-}1914)}{5040v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$H_1=\frac{(882v^4+4656v^3+8751v^2+6891v+1914)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$I_1=\frac{(1596v^6-2142v^5-14226v^4+7791v^3+32835v^2+3108v\text{-}5868)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$J_1=\frac{(1764v^4+492v^3-13818v^2-12558v+5868)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$K_1=\frac{-(546v^6+1008v^5-3684v^4-10503v^3-7014v^2+369v+1026)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$L_1=\frac{-(882v^4-4164v^3+3891v^2+2331v\text{-}1026)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$M_1=\frac{(112v^6+546v^5+886v^4+481v^3-41v^2-70v)}{5040(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$E_2=\frac{-(5040v^5-15120v^4-25200v^3+75600v^2+20160v\text{-}60480)}{(5040v-5040)(v-2)(v+2)(v-3)}-\frac{-(5040v^6+15120v^5+25200v^4-75600v^3-20160v^2+60480v)}{5040(v-1)(v-2)(v+2)(v-3)}$$

$$F_2=\frac{(-5040v^5+20160v^4+5040v^3-80640v^2+60480v)}{(5040v-5040)(v-2)(v+2)(v-3)}-\frac{-(5040v^6+15120v^5+25200v^4-75600v^3-20160v^2+60480v)}{5040v(v-1)(v-2)(v+2)(v-3)}$$

$$G_2=\frac{-(3v^9-45v^8+264v^7-714v^6+144v^5+3678v^4-6314v^3+999v^8+2963v+978)}{(5040v-5040)(v-2)(v+2)(v-3)}-\frac{(-1358v^6+4956v^5+2944v^4-19025v^3+3268v^2+11129v\text{-}1914v\text{-}1914)}{5040v(v-1)(v-2)(v+2)(v-3)}$$

$$H_2=\frac{-(-9v^8-6v^7+150v^6+84v^5-756v^4-126v^3+2778v^2+3189v+978)}{(5040v-5040)(v-2)(v+2)(v-3)}-\frac{(882v^4+4656v^3+7851v^2+6891v+1914)}{5040v(v-1)(v-2)(v+2)(v-3)}$$

$$I_2=\frac{(1596v^6-2142v^5-14226v^4+7791v^3+32835v^2+3108v\text{-}5868)}{5040v(v-1)(v-2)(v+2)(v-3)}-\frac{(-9v^9+84v^8-66v^7-1788v^6+2784v^5+10782v^4-11652v^3-21525v^2+8592v+6516)}{(5040v-5040)(v-2)(v+2)(v-3)}$$

$$\frac{(-18v\ \ +108v^7+180v^6-1872v^5+948v^4+8628v^3-7944v\quad 13062v+6516)}{(5040v-5040)(v-2)(v+2)(v-3)}$$

$$\frac{1764\ ^4\quad 492\ ^3\quad 13818v\ \text{-}12558v+5868)}{5040v(v-1)(v-2)(\ \ +2)(v\quad 3)}$$

$$K_2=\frac{(-9v^9-33v^8-240v^7+438v^6+2100v^5-798v^4-6354v^3-3747v^2+1227v+882)}{(5040v-5040)(v-2)(v+2)(v-3)}-\frac{(546v^6+1008v^5\text{-}3684v^4-10503v^3-7014v^2+369v+1026)}{5040v(v-1)(v-2)(v+2)(v-3)}$$

$$L_2=\frac{-(9v^8+114v^7\text{-}510v^6+744v^5+1104v^4-4506v^3-3378v^2+1149v\text{-}882)}{(5040v-5040)(v-2)(v+2)(v-3)}-\frac{(882v^4-4146v^3+3891v^2+2331v\text{-}1026)}{5040v(v-1)(v-2)(v+2)(v-3)}$$

$$M_2=\frac{(3v^9+6v^8-42v^7\text{-}84v^6+168v^5+462v^4+220v^3-93v^2-58v)}{(5040v-5040)(v-2)(v+2)(v-3)}+\frac{(112v^6+546v^5+886v^4+481v^3-41v^2\text{-}70v)}{5040v(v-1)(v-2)(v+2)(v-3)}$$

$$E_3=\frac{(-5040v^5+20160v^4+5040v^3-80640v^2+60480v)}{5040v(v-1)(v-2)(v+2)(v-3)}-\frac{(-5040v^6+15120v^5+25200v^4-75600v^3-20160v^2+60480v)}{5040v(v-1)(v-2)(v+2)(v-3)}$$

$$F_3=\frac{(-2520v^6+7560v^5+12600v^4-37800v^3-10080v^2+30240v)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}-\frac{(-5040v^6+15120v^5+25200v^4-75600v^3-20160v^2+60480v)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$G_3=\frac{-(-210v^6+756v^5+672v^4-3935v^3+1980v^8+1319v\text{-}582)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}-\frac{(-1358v^6+4956v^5+2944v^4-19025v^3+3268v^2+11129v\text{-}1914)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$H_3=\frac{-(126v^4+1008v^3+2433v^2+2133v+582)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}-\frac{(882v^4+4656v^3+8751v^2+6891v+1914)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$I_3=\frac{(1596v^6\text{-}2142v^5\text{-}14226v^4+7791v^3+32835v^2+3108v\text{-}5868)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}-\frac{(2100v^6+5922v^5+10878v^4-27633v^3-10173v^2+15780v+1044)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$J_3=\frac{(252v^4+756v^3-2694v^2-5874v+1044)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}+\frac{(1764v^4+492v^3-13818v^2\text{-}12558v+5868)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$k_3=\frac{-(-210v^6+1008v^5+1428v^4-6009v^3-5754v^2+1743v+1278)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}-\frac{(546v^6+1008v^5\text{-}3684v^4-10503v^3-7014v^2+369v+1026)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$L_3=\frac{(-126v^4+252v^3+1347v^2-3333v+1278)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}-\frac{(882v^4-4146v^3+3891v^2+2331v\text{-}1026)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$M_3=\frac{(112v^6+546v^5+886v^4+481v^3-41v^2-70v)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}+\frac{(126v^5+378v^4+233v^3-87v^2\text{-}58v)}{2520v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$E_4=\frac{(-2520v^6+7560v^5+12600v^4-37800v^3-10080v^2+30240v)}{840v(v-1)(v+1)(v-2)(v+2)(v-3)}-\frac{-(5040v^6+15120v^5+25200v^4-75600v^3-20160v^2+60480v)}{1680v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$F_4=\frac{-(5040v^5-25200v^4+25200v^3+25200v^2-30240v)}{5040v(v-1)(v-2)(v-3)}-\frac{-(5040v^6+15120v^5+25200v^4-75600v^3-20160v^2+60480v)}{5040v(v-1)(v+1)(v-2)(v-3)}$$

$$G_4=\frac{-(3v^9\text{-}45v^8+270v^7\text{-}810v^6+780v^5+1842v^4-4430v^3+1455v^8+1517v\text{-}582)}{5040v(v-1)(v-2)(v-3)}-\frac{(-1358v^6+4956v^5+2944v^4-19025v^3+3268v^2+11129v\text{-}1914v-1914)}{5040v(v-1)(v+1)(v-2)(v+3)(v-3)}$$

$$H_4=\frac{-(-9v^8\text{-}12v^7+126v^6+168v^5-420v^4-210v^3+1602v^2+1935v+582)}{5040v(v-1)(v-2)(v-3)}-\frac{(882v^4+4656v^3+8751v^2+6891v+1914)}{5040v(v-1)(v+1)(v-2)(v-3)}$$

$$I_4=\frac{(1596v^6-2142v^5-14226v^4+7791v^3+32835v^2+3108v\text{-}5868)}{5040v(v-1)(v+1)(v-2)(v-3)}-\frac{(-9v^9+84v^8-78v^7-1716v^6+3312v^5+8994v^4+14820v^3-14157v^2+13584v+1044)}{5040v(v-1)(v+1)(v-2)(v-3)}$$

$$J_4=\frac{(1764v^4+492v^3-13818v^2-12558v+5868)}{5040v(v-1)(v+1)(v-2)(v-3)}-\frac{(-18v^8-96v^7-252v^6+1344v^5+840v^4-5460v^3+576v^2+8070v-1044)}{5040v(v-1)(v-2)(v-3)}$$

$$K_4=\frac{-(9v^9-33v^8-234v^7+462v^6+2016v^5-1134v^4-6270v^3-2571v^2+2481v+1278)}{5040v(v-1)(v-2)(v-3)}-\frac{(546v^6+1008v^5-3684v^4-10503v^3-7014v^2+369v+1026)}{5040v(v-1)(v+1)(v-2)(v-3)}$$

$$L_4=\frac{-(9v^8+108v^7-414v^6+108v^5+2940v^4-6390v^3+2922v^2+2595v\text{-}1278)}{5040v(v-1)(v-2)(v-3)}-\frac{(882v^4-4164v^3+3891v^2+2331v-1026)}{5040v(v-1)(v+1)(v-2)(v-3)}$$

$$M_4=\frac{(3v^9+6v^8-42v^7-84v^6+168v^5+462v^4+220v^3-93v^2-58v)}{5040v(v-1)(v-2)(v-3)}-\frac{(112v^6+546v^5+882v^4+481v^3-41v^2-70v)}{5040v(v-1)(v+1)(v-2)(v-3)}$$

$$F_5=\frac{(-1680v^6+5040v^5+8400v^4-25200v^3-6720v^2+20160v)}{840v(v-1)(v-2)(v-3)}-\frac{(-5040v^6+15120v^5+25200v^4-75600v^3-20160v^2+60480v)}{1680v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$G_5 = \frac{(140v^6 - 546v^5 - 112v^4 + 1625v^3 + 50v^8 - 1739v + 582)}{840v(v-1)(v+1)(v-2)(v+2)(v-3)} - \frac{(-1358v^6 + 4956v^5 + 2944v^4 - 19025v^3 + 3268v^2 + 11129v - 1914)}{1680v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$H_5 = \frac{(126v^4 + 588v^3 + 1173v^2 + 1293v + 582)}{840v(v-1)(v+1)(v-2)(v+2)(v-3)} - \frac{(882v^4 + 4656v^3 + 8751v^2 + 6891v + 1914)}{1680v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$I_5 = \frac{(1470v^6 - 4032v^5 - 8358v^4 + 20283v^3 + 9543v^2 - 16620v + 1476)}{840v(v-1)(v+1)(v-2)(v+2)(v-3)} + \frac{(1596v^6 - 2142v^5 - 14226v^4 + 7791v^3 + 32835v^2 + 3108v - 5868)}{1680v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$J_5 = \frac{(1764v^4 + 492v^3 - 13818v^2 - 12558v + 5868)}{1680v(v-1)(v+1)(v-2)(v+2)(v-3)} - \frac{-(252v^4 + 84v^3 + 1434v^2 + 1254v + 1476)}{840v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$K_5 = \frac{(840v^6 - 2898v^5 - 3948v^4 + 15039v^3 + 3864v^2 - 10143v + 1242)}{840v(v-1)(v+1)(v-2)(v+2)(v-3)} - \frac{(546v^6 + 1008v^5 - 3684v^4 - 10503v^3 - 7014v^2 + 369v + 1026)}{1680v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$L_5 = \frac{-(126v^4 - 672v^3 + 1173v^2 - 1287v + 1242)}{840v(v-1)(v+1)(v-2)(v+2)(v-3)} - \frac{(-882v^4 - 4146v^3 + 3891v^2 + 2331v - 1026)}{1680v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

$$M_5 = \frac{(112v^6 + 546v^5 + 886v^4 + 481v^3 - 41v^2 - 70v)}{1680v(v-1)(v+1)(v-2)(v+2)(v-3)} - \frac{(-70v^6 + 84v^5 + 182v^4 - 853v^3 - 403v^2 + 478v)}{840v(v-1)(v+1)(v-2)(v+2)(v-3)}$$

The derivative of the generalized hybrid block (7) gives

$$y'_{n+v} = y'_n + h(N_0 f_n + O_0 f_{n+v} + Q_0 f_{n+v+1} + R_0 f_{n+2} + S_0 f_{n+v+2} + T_0 f_{n+3})$$

$$y'_{n+1} = y'_n + h(N_1 f_n + O_1 f_{n+v} + Q_1 f_{n+v+1} + R_1 f_{n+2} + S_1 f_{n+v+2} + T_1 f_{n+3})$$

$$y'_{n+v+1} = y'_n + h(N_2 f_n + O_2 f_{n+v} + Q_2 f_{n+v+1} + R_2 f_{n+2} + S_2 f_{n+v+2} + T_2 f_{n+3})$$

$$y'_{n+2} = y'_n + h(N_3 f_n + O_3 f_{n+v} + Q_3 f_{n+v+1} + R_3 f_{n+2} + S_3 f_{n+v+2} + T_3 f_{n+3})$$

$$y'_{n+v+2} = y'_n + h(N_4 f_n + O_4 f_{n+v} + Q_4 f_{n+v+1} + R_4 f_{n+2} + S_4 f_{n+v+2} + T_4 f_{n+3})$$

$$y'_{n+3} = y'_n + h(N_5 f_n + O_5 f_{n+v} + Q_5 f_{n+v+1} + R_5 f_{n+2} + S_5 f_{n+v+2} + T_5 f_{n+3})$$

Where

$$N_0 = \frac{v}{2520(v+1)(v+2)}(-3v^5 + 21v^4 - 21v^3 - 105v^2 + 980v + 2520)$$

$$O_0 = \frac{-v}{840(v-1)(v-2)(v-3)}(-4v^5 + 56v^3 + 315v^2 - 1820v + 2520)$$

$$P_0 = \frac{v^2}{840(v^2-1)}(3v^4 - 14v^3 - 42v^2 + 280v + 840)$$

$$Q_0 = \frac{v^2}{210(v-1)(v+1)(v-2)}(-2v^4 + 7v^3 + 49v^2 - 280v + 420)$$

$$R_0 = \frac{-v^2}{840(v-1)(v-2)}(3v^4 - 7v^3 - 63v^2 + 35v + 420)$$

$$S_0 = \frac{1}{840(v-1)(v+1)(v+2)}(4v^4 - 28v^3 + 28v^2 + 175v - 420)$$

$$T_0 = \frac{v^3}{2520(v-1)(v-2)(v-3)}(3v^4 - 42v^2 + 280)$$

$$N_1 = \frac{1}{2520v(v+1)(v+2)}(945v^3 + 2037v^2 + 651v - 241)$$

$$O_1 = \frac{-1}{840v(v-1)(v-2)(v-3)}(266v^2 + 560v + 241)$$

$$P_1 = \frac{-1}{840v(v^2-1)}(-655v^3 - 798v^2 + 224v + 172)$$

$$Q_1 = \frac{1}{210v(v-1)(v+1)(v-2)}(133v^2 + 147v - 86)$$

$$R_1 = \frac{1}{840v(v-1)(v-2)}(-175v^3 - 273v^2 + 7v + 53)$$

$$S_1 = \frac{-1}{840v(v-1)(v+1)(v+2)}(266v^2 + 28v - 53)$$

$$T_1 = \frac{1}{2520(v-1)(v-2)(v-3)}(105v^3 + 168v^2 - 32)$$

$$N_2 = \frac{v+1}{2520v(v+2)}(-3v^5 + 27v^4 - 27v^3 + 12v^2 + 1133v - 241)$$

$$O_2 = \frac{-(v+1)^3}{840v(v-1)(v-2)(v-3)}(-4v^4 + 12v^3 + 32v^2 - 163v + 241)$$

$$P_2 = \frac{(v+1)^2}{840v(v-1)}(3v^4 - 23v^3 + 18v^2 + 292v - 172)$$

$$Q_2 = \frac{v+1}{210v(v-1)(v-2)}(-2v^5 + 11v^4 + 29v^3 - 209v^2 + 319v - 86)$$

$$R_2 = \frac{(v+1)^3}{840v(v-1)(v-2)}(-3v^4 + 16v^3 + 24v^2 - 152v + 53)$$

$$S_2 = \frac{(v+1)^2}{840v(v-1)(v-2)}(4v^4 - 40v^3 + 136v^2 - 187v + 53)$$

$$T_2 = \frac{-(v+1)^3}{2520v(v-1)(v-2)(v-3)}(-3v^4 + 9v^3 + 24v^2 - 96v + 32)$$

$$N_3 = \frac{1}{315v(v+1)(v+2)}(105v^3 + 273v^2 + 84v - 34)$$

$$O_3 = \frac{-2}{105v(v-1)(v-2)(v-3)}(7v^2 + 35v + 17)$$

$$Q_3 = \frac{4}{105v(v-1)(v+1)(v-2)}(7v^2 + 28v - 4)$$

$$Q_3 = \frac{4}{105v(v-1)(v+1)(v-2)}(7v^2 + 28v - 4)$$

$$S_3 = \frac{-2}{105v(v-1)(v+1)(v+2)}(7v^2+21v-11)$$

$$S_3 = \frac{-2}{105v(v-1)(v+1)(v+2)}(7v^2+21v-11)$$

$$T_3 = \frac{2}{315(v-1)(v-2)(v-3)}(21v^2-4)$$

$$N_4 = \frac{(v+2)}{2520v(v+1)}\left(-3v^5+33v^4-141v^3+327v^2+236v-68\right)$$

$$O_4 = \frac{-(v+2)^3}{840v(v-1)(v-2)(v-3)}\left(-4v^4+24v^3-40v^2+19v+34\right)$$

$$P_4 = \frac{-(v+2)^3}{840v(V^2-1)}\left(-3v^4+32v^3-114v^2+44v+8\right)$$

$$Q_4 = \frac{-(v+2)^3}{210v(v-1)(v+1)(v-2)}\left(-2v^4+19v^3-41v^2+34v+4\right)$$

$$R_4 = \frac{-(v+2)^3}{840v(v-1)(v-2)}\left(3v^4-25v^3+51v^2+5v-22\right)$$

$$S_4 = \frac{-(v+2)}{840v(v-1)(v+1)}\left(-4v^5+44v^4-188v^3+121v^2+128v-44\right)$$

$$T_4 = \frac{(v+2)^3}{2520(v-1)(v-2)(v-3)}\left(3v^4-18v^3+30v^2+12v-8\right)$$

$$N_5 = \frac{3}{280v(v+1)(v+2)}\left(35v^3+63v^2+49v-19\right)$$

$$O_5 = \frac{-9}{280v(v-1)(v-2)(v-3)}\left(14v^2+19\right)$$

$$P_5 = \frac{9}{280v(v^2-1)}\left(35v^3+42v^2-56v+12\right)$$

$$R_5 = \frac{9}{280v(v-1)(v-2)}\left(35v^3-147v^2+133v-33\right)$$

$$R_5 = \frac{9}{280v(v-1)(v-2)}\left(35v^3-147v^2+133v-33\right)$$

$$S_5 = \frac{-9}{280v(v-1)(v+1)(v-2)}\left(14v^2-28v+33\right)$$

$$T_5 = \frac{3}{280(v-1)(v-2)(v-3)}\left(35v^3-168v^2+280v-128\right)$$

When $v=\frac{1}{4}$ This produces the block with its derivative

$$y_{n+\frac{1}{4}} = y_n + \frac{h}{4}y_n' + h^2\begin{pmatrix}\frac{802033}{46448640}f_n + \frac{327721}{19869696}f_{n+\frac{1}{4}} - \frac{49039}{5160960}f_{n+1} + \frac{1829}{215040}f_{n+\frac{5}{4}} \\ -\frac{38809}{12042240}f_{n+2} + \frac{4117}{2322432}f_{n+\frac{9}{4}} - \frac{10909}{170311680}f_{n+3}\end{pmatrix}$$

$$y_{n+1} = y_n + hy_n' + h^2\begin{pmatrix}\frac{1403}{22680}f_n + \frac{1154}{3465}f_{n+\frac{1}{4}} + \frac{293}{1260}f_{n+1} - \frac{36}{245}f_{n+\frac{5}{4}} + \frac{11}{280}f_{n+2} \\ -\frac{58}{2853}f_{n+\frac{9}{4}} + \frac{97}{145530}f_{n+3}\end{pmatrix}$$

$$y_{n+\frac{5}{4}} = y_n + \frac{5h}{4}y_n' + h^2\begin{pmatrix}\frac{659525}{9289728}f_n + \frac{3071875}{6623232}f_{n+\frac{1}{4}} + \frac{160375}{344064}f_{n+1} - \frac{229675}{903168}f_{n+\frac{5}{4}} \\ +\frac{505625}{7225344}f_{n+2} - \frac{12125}{331776}f_{n+\frac{9}{4}} + \frac{285625}{238436352}f_{n+3}\end{pmatrix}$$

$$y_{n+2} = y_n + 2h_n' + h^2\begin{pmatrix}\frac{268}{2835}f_n + \frac{2336}{2695}f_{n+\frac{1}{4}} + \frac{38}{35}f_{n+1} - \frac{64}{315}f_{n+\frac{5}{4}} + \frac{122}{441}f_{n+2} \\ -\frac{352}{2835}f_{n+\frac{9}{4}} + \frac{38}{10395}f_{n+3}\end{pmatrix}$$

$$y_{n+\frac{9}{4}} = y_n + \frac{9h}{4}y_n' + h^2\begin{pmatrix}\frac{57969}{573440}f_n + \frac{1008207}{1003520}f_{n+\frac{1}{4}} + \frac{723897}{573440}f_{n+1} - \frac{63423}{501760}f_{n+\frac{5}{4}} \\ +\frac{1799901}{4014080}f_{n+2} - \frac{23463}{143360}f_{n+\frac{9}{4}} + \frac{19197}{4014080}f_{n+3}\end{pmatrix}$$

$$y_{n+3} = y_n + 3hy_n' + h^2\begin{pmatrix}\frac{29}{280}f_n + \frac{3942}{2695}f_{n+\frac{1}{4}} + \frac{54}{35}f_{n+1} + \frac{108}{245}f_{n+\frac{5}{4}} + \frac{999}{1960}f_{n+2} \\ +\frac{2}{5}f_{n+\frac{9}{4}} + \frac{87}{2156}f_{n+3}\end{pmatrix}$$

$$y'_{n+\frac{1}{4}} = y_n' + \frac{h}{447068160}\begin{pmatrix}43495529f_n + 76882032f_{n+\frac{1}{4}} - 32187771f_{n+1} + 28641888f_{n+\frac{5}{4}} \\ -10763973f_{n+2} + 5912368f_{n+\frac{9}{4}} - 213033f_{n+3}\end{pmatrix}$$

$$y_{n+1} = y_n' + \frac{h}{1746360}\begin{pmatrix}62909f_n + 916128f_{n+\frac{1}{4}} + 1487871f_{n+1} - 830016f_{n+\frac{5}{4}} \\ +221463f_{n+2} - 115808f_{n+\frac{9}{4}} + 3813f_{n+3}\end{pmatrix}$$

$$y_{n+\frac{5}{4}} = y_n' + \frac{h}{89413632}\begin{pmatrix}3309845f_n + 46638000f_{n+\frac{1}{4}} + 87127425f_{n+1} - 30523680f_{n+\frac{5}{4}} \\ +10605375f_{n+2} - 5574800f_{n+\frac{9}{4}} + 184875f_{n+3}\end{pmatrix}$$

$$y'_{n+2} = y_n' + \frac{h}{218295}\begin{pmatrix}5621f_n + 120672f_{n+\frac{1}{4}} + 152460f_{n+1} + 69696f_{n+\frac{5}{4}} \\ +128997f_{n+2} - 41888f_{n+\frac{9}{4}} + 1032f_{n+3}\end{pmatrix}$$

$$y_{n+2} = y_n' + \frac{h}{64680}\begin{pmatrix}-2233f_n + 45792f_{n+\frac{1}{4}} - 10395f_{n+1} + 95040f_{n+\frac{5}{4}} \\ -53163f_{n+2} + 105952f_{n+\frac{9}{4}} + 13047f_{n+3}\end{pmatrix}$$

$$y_{n+2} = y_n' + \frac{h}{64680}\begin{pmatrix}-2233f_n + 45792f_{n+\frac{1}{4}} - 10395f_{n+1} + 95040f_{n+\frac{5}{4}} \\ -53163f_{n+2} + 105952f_{n+\frac{9}{4}} + 13047f_{n+3}\end{pmatrix}$$

Test Problems

The following second order initial value problems of ODEs are considered in order to examine the accuracy of the new developed method.

Problem 1: $y''-100y=0$, $y(0)=1$, $y'(0)=-10$, $h=0.01$

Exact Solution: $y(x)=e^{-10x}$

Problem 2: $y''-x(y')^2=0$, $y(0)=1$, $y'(0)=$ $y(x)=1-$, $h=0.003125$

Exact Solution: $y(x)=1+\frac{1}{2}In\left[\frac{2+x}{2-x}\right]$

Problem 3: $y''=y'$, $y(0)=0$, $y'(0)=-1$, $h=0.01$

Exact Solution: $y(x)=1-e^{x}$

Table 1: Comparison of the New Hybrid Block Method with Uniform Accurate Block Integrators (Awari et al., 2014) and Zero Stable Continuous Block Method (Awari & Abada, 2014) for Solving Problem 1.

x-values	Exact Solution	Computed Solution	Error in New method, *k*=3	Error in Awari et al. (2014) *k*=6 , *h*=0.01	Error in Awari & Abada (2014) *k*=7, h=0.01
0.1	0.367879441171442330	0.367879441282827010	1.113847E-10	1.353E-07	1.440E-08
0.2	0.135335283236612730	0.135335283744420470	5.078077E-10	3.658E-07	3.850E-08
0.3	0.049787068367863944	0.049787069884009189	1.516145E-09	6.051E-07	6.330E-08
0.4	0.018315638888734165	0.018315643082197818	4.193464E-09	8.502E-07	8.800E-08
0.5	0.006737946999085461	0.006737958432813206	1.143373E-08	1.104E-06	1.151E-07
0.6	0.002478752176666354	0.002478783272535241	3.109587E-08	1.369E-06	1.427E-07
0.7	0.000911881965554514	0.000911966499798363	8.453424E-08	1.450E-06	1.716E-07
0.8	0.000335462627902511	0.000335692418749578	2.297908E-07	1.597E-06	1.796E-07
0.9	0.000123409804086679	0.000124034441604693	6.246375E-07	1.763E-06	1.941E-07
0.1	0.000045399929762485	0.000047097871085910	1.697941E-06	1.946E-06	2.109E-07

Table 2: Comparison of the New Hybrid Block Method with Block Method (Kuboye, 2015), Numerical Methods (Adeniyi & Alabi, 2011) where two Continuous Collocation Methods for *k*=6 were considered for Solving Problem 2.

x-values	Exact Solution	Computed Solution	Error in new Method, *k*=3,	Error in Kuboye (2015) Method, *k*=6, *h*=0.1	Error in Adeniyi and Alabi (2011) *k*=6, *h*=0.1	Error in Adeniyi and Alabi (2011) *k*=6, *h*=0.1
0.1	1.050041729278491400	1.050041729279076500	5.850875E-13	9.577668E-10	0.1329867326E-09	0.1708719055E-09
0.2	1.100335347731075600	1.100335347733924400	2.848832E-12	2.368709E-09	0.5872691257E-08	0.6836010114E-08
0.3	1.151140435936466800	1.151140435930138000	6.328715E-12	3.732243E-09	0.1327845616E-07	0.1555757709E-07
0.4	1.202732554054082100	1.202732547297689700	6.756392E-09	5.475119E-09	0.2317829012E-07	0.2880198295E-07
0.5	1.255412811882995200	1.255412798081808400	1.380119E-08	1.142189E-08	0.3218793564E-07	0.4802328029E-07
0.6	1.309519604203111900	1.309519582454941500	2.174817E-08	4.567944E-08	0.6871246012E-07	0.7628531256E-07
0.7	1.365443754271396400	1.365443646966158600	1.073052E-07	2.055838E-06	0.1012728156E-06	0.1157914170E-06
0.8	1.423648930193601700	1.423648730059607500	2.001340E-07	4.248299E-06	0.1231093271E-06	0.1727046080E-06
0.9	1.484700278594052000	1.484699969755750100	3.088383E-07	6.660458E-06	0.2019286712E-06	0.2561456831E-06
1.0	1.549306144334054800	1.549305163826689900	9.805074E-07	9.445166E-06	0.2990871645E-06	0.3815695118E-06

Table 3: Comparison of the New Hybrid Block Method with Block Method (Kuboye, 2015), Block Hybrid Backward Difference Formula (Mohammed & Adeniyi, 2014) and Block Method (Mohammed, 2011) for Solving Problem 3.

x-values	Exact Solution	Computed Solution	Error in new Method, *k*=3 *h*=0.1	Error in Kuboye (2015) Method, *k*=5 *h*=0.1	Error in Mohammed and Adeniyi (2014), *k*=5, *h*=0.1	Error in Mohammed (2011), *k*=5, *h*=0.1
0.1	-0.105170918075647710	-0.105170918075644850	2.858824E-15	2.508826E-13	2.004000000E-07	2.198000000E-05
0.2	-0.221402758160169850	-0.221402758158730170	1.439682E-12	6.493175E-11	5.386000000E-07	6.070400000E-06
0.3	-0.349858807576003180	-0.349858807520089350	5.591383E-11	1.683146E-09	8.840000000E-07	1.005100000E-05
0.4	-0.491824697641270350	-0.491824692844668230	4.796602E-09	1.700635E-08	1.229700000E-06	1.402530000E-05
0.5	-0.648721270700128190	-0.648721260662319610	1.003781E-08	1.025454E-07	1.575200000E-06	1.799340000E-05
0.6	-0.822118800390508890	-0.822118784488882760	1.590163E-08	2.558711E-06	1.920400000E-06	2.161620000E-05
0.7	-1.013752707470476600	-1.013752678770335700	2.870014E-08	5.273300E-06	2.506000000E-06	2.799300000E-05
0.8	-1.225540928492467900	-1.225540885645167900	4.284730E-08	8.275935E-06	3.106000000E-06	3.456100000E-05
0.9	-1.459603111156949900	-1.459603052578256800	5.857869E-08	1.161667E-05	3.705000000E-06	4.111400000E-05
0.1	-1.718281828459045500	-1.718281743966074900	8.449297E-08	1.542187E-05	4.304000000E-06	4.765600000E-05

Discussion of Result

It is apparent in Tables 1 and 2 that the results of the new hybrid block method *k*=3 outperform Awari et al. [10] *k*=6, Awari and Abada [11] *k*=7, Kuboye (2015) *k*=6 and Adeniyi and Alabi [12] *k*=6 for solving Problems 1 and 2 despite the high step-lengths *k* involved in these methods. Furthermore, in Table 3, the results of the new block method *k*=3 are better when compared with Kuboye et al. [13] *k*=5, Mohammed and Adeniyi [14] *k*=5 and Mohammed [15] *k*=5 for solving Problem 2.

Conclusion

The derivation of block method with three generalised hybrid points through interpolation and collocation approach for solving second order initial value problems of ODEs has been examined in this paper. In order to compare the new developed method with the existing ones, a specific hybrid point was selected and the results generated compared favourably with existing methods in terms of accuracy. These are evidently shown in Tables 1-3.

References

1. Lambert JD (1973) Computational Methods in Ordinary Differential Equations. Introductory Mathematics for Scientists and Engineers. Wiley Online Library.
2. Fatunla SO (1988) Numerical Methods for Initial Value Problems in Ordinary Differential Equation. Academic Press Inc. Harcourt Brace Jovanovich Publishers, New York, USA.

3. Brugnano L, Trigiante D (1998) Solving Differential Equations by Multistep Initial and Boundary Value Methods. CRC Press, USA.
4. Awoyemi DO (2001) A new sixth-order algorithm for general second order ordinary differential equations. Int J Comput Math 77: 117-124.
5. Jator SN (2007) A sixth order linear multistep method for the direct solution of y"= f (x, y, y'). Int J Pure Appl Math 40: 457- 472.
6. Kayode SJ, Adeyeye O (2011) A 3-Step Hybrid Methods for Direct Solution of Second Order Initial Value Problems. Aust Basic Applied Sci 5: 2121-2126.
7. Odejide SA, Adeniran AO (2012) A Hybrid linear collocation multistep scheme for solving first order initial value problems. J Nigerian Math Soc 31: 229-241.
8. Sagir AM (2013) An accurate computation of block hybrid method for solving stiff ordinary differential equations. World Acad Sci Eng Technol 76: 384-387.
9. Yap KY, Ismail F, Senu N (2014) An accurate block hybrid collocation method for third order ordinary differential equations. J Appl Math 1-9.
10. Awari YS, Abada AA (2014) A Class of Seven Point Zero Stable Continuous Block Method for Solution of Second Order Ordinary Differential Equation. IJMSI 2: 47-54.
11. Awari YS, Chima EE, Kamoh NM, Oladele FL (2014) A Family of Implicit Uniformly Accurate Order Block Integrators for the Solution of *y*" = *f* (*x,y,y'*). IJMSI 2: 33-46.
12. Adeniyi RB, Alabi M (2011) A collocation method for direct numerical integration of initial value problems in higher order ordinary differential equations. Alexandru Ioan Cuza 57: 311-321.
13. Kuboye JO (2015) Block methods for solving higher order ordinary differential equations directly using interpolation and collocation approach. Universiti Utara Malaysia.
14. Mohammed U, Adeniyi RB (2014) Derivation of Five Step Block Hybrid Backward Differential Formulas (HBDF) through the Continuous Multi-Step Collocation for Solving Second Order Differential Equation. Pac J Sci Technol 15: 89-95.
15. Mohammed U (2011) A class of implicit five-step block method for general second order ordinary differential equations. JNMS 30: 25-39.

Author Affiliation

Top

[1]*Department of Mathematics, Faculty of Sciences, Federal University Oye-Ekiti, Ekiti-State, Nigeria*

[2]*Department of Mathematics, School of Quantitative Sciences, Universiti Utara Malaysia (UUM), Malaysia*

[3]*Al Jouf University, Saudi Arabia*