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Green's Function of the Wave Equation for a Fractured Dissipative HTI Medium Taking the Viscoelasticity of the System into Account

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Abstract

In this paper we derive the Green's function of the wave equation for a fractured dissipative HTI medium. Inside the fractures there is a viscous fluid which adds to the attenuation of the wave. Previous works have been done for the elastic medium where the stiffness tensor have all real components. In this scenario the host rock and the fluid inside the fractures both have viscoelastic properties. Thus, complex terms in the stiffness tensor has been introduced to account for this viscoelasticity. Finally, we arrive to a Green Christoffel type of equation with additional complex terms due to the introduction of viscoelasticity. We then perform a Fourier Transform to solve for the Green's function and finally an Inverse Fourier Transform to obtain the Green's function in (x,t) space. This Green's function can be used to determine how a wave passing through a viscoelastic layer (e.g. hydrocarbon layer) is changed after passing through it. Thus, in turn it can be used to detect hydrocarbon layers.

Keywords

Green's function; Wave equation; Viscoelasticity

Introduction

Green's function is used in mathematics to solve inhomogeneous boundary value problems. In the context of this paper, to get an idea about its physical interpretation, we can define Green's function as the response of the medium on the force action. To elucidate this further let's take the example of a drum and a stick. When the stick strikes the drum, it vibrates and produces sound. Now, if the same drum is struck by a different object such as a hammer it vibrates differently, producing a different sound. This is where the Green's function comes into picture. If we can construct a function that contains all the properties of the system (in this case the drum) such that irrespective of the object that strikes the system (drum), we can determine the response of the system. We only need to know the force exerted by the object (stick or hammer) on the system. Thus, one property of the Green's function we can observe from here is that it is independent of the force on the system.

In Bretin and Wahab, 2011 [1] and Vavrycuk, 2007 [2] they have derived the Green's function for the anisotropic viscoelastic media. General stiffness tensor for the orthorhombic and TI medium is used with no involvement of specific medium related stiffness tensor that properly reflect the characteristics of a viscoelastic rock.

Previous work by Vshivtsev et al., 1993 [3] have derived the Green's function of the wave equation for an Elastic and Anisotropic media. Here in this paper we have followed almost the same process except now viscoelasticity in the host rocks as well as in the fractures has been introduced. Thus, the Stiffness Tensor now has complex terms Chichinina et al., 2006 [4]. The derivation is done step by step in the following sections as simply is possible (Figure 1).

Theory: (The formulation of the problem)

We know the normal Hooke's law for the Elastic case as:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} \tag{1}$$

Where σ is the stress, C_{ijkl} the Stiffness tensor which is a 4[th] rank tensor and ε is the strain. Now if we have viscosity in our system we have an added term in the Hooke's law equation.

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} + \eta_{ijkl}\frac{\partial \varepsilon_{kl}}{\partial t} \tag{2}$$

Here η_{ijkl} is the 4[th] rank Viscosity tensor and 't' is time.

From continuum mechanics we have the wave equation as follows:

$$\rho\frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} \tag{3}$$

Here 'ρ' is the density of the medium and 'u' is the displacement of a particle of the medium through which the wave passes.

Let us now we differentiate w.r.t. x_j

$$\frac{\partial \sigma_{ij}}{\partial x_j} = C_{ijkl}\frac{\partial^2 u_k}{\partial x_j \partial x_l} + \varepsilon_{kl}\frac{\partial C_{ijkl}}{\partial x_j \partial x_l} + \eta_{ijkl}\frac{\partial^2 \varepsilon_{kl}}{\partial x_j \partial t} + \frac{\partial \varepsilon_{kl}}{\partial t}\frac{\partial \eta_{ijkl}}{\partial x_j} \tag{4}$$

C_{ijkl} and η_{ijkl} does not change with x_j, thus the derivative w.r.t. x_j gives 0.

Strain can be written in the form of derivative of displacement as follows:

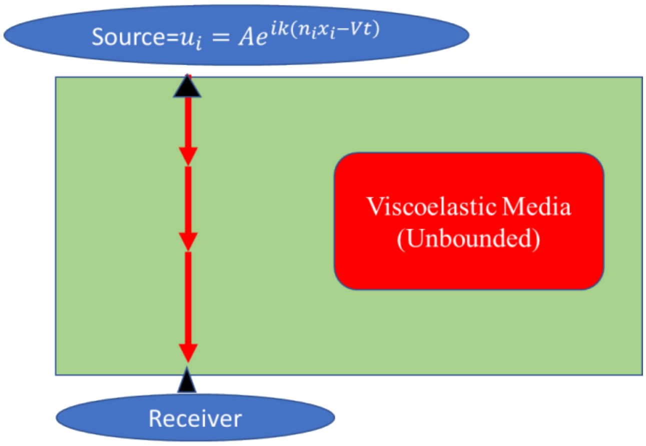

Figure 1: The Problem.

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$$\varepsilon_{kl} = \frac{1}{2}\left[\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right] \quad (5)$$

Substituting in and we get the *Modified Wave Equation*

$$\rho\frac{\partial^2 u_i}{\partial t^2} = C_{ijkl}\frac{\partial^2 u_k}{\partial x_j \partial x_l} + \eta_{ijkl}\frac{\partial^3 u_{kl}}{\partial x_j \partial x_l \partial t} \quad (6)$$

Let's consider a Plane Wave passing through the medium:

$$u_i = Ae^{ik(n_i x_i - Vt)} \quad (7)$$

Substituting the Plane Wave equation in we get,

$$\left(\Gamma_{ik} - i\omega D_{ik} - \rho V^2 \delta_{ik}\right)u_k = 0 \quad (8)$$

This is the *Modified Green Christoffel Equation*

Where, V is the Phase Velocity and thus $V = \frac{\omega}{k} \Rightarrow kV = \omega$, And $U_i = U_k \delta_{ik}$ (Internal Tensor Product). And $\Gamma_{ik} = C_{ijkl} n_j n_l$, $D_{ik} = \eta_{ijkl} n_j n_l$.

Solving for Green's function:

$$\left(\rho\delta_{ik}\frac{\partial^2}{\partial t^2} - C_{ijkl}\nabla_j\nabla_l - \eta_{ijkl}\nabla_j\nabla_l\frac{\partial}{\partial t}\right)\tilde{G}_{km}(t,x) = \delta_{im}\delta(t)\delta^3(x) \quad (9)$$

Now Converting to Fourier Space:

$$\tilde{G}_{km}(t,x) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{-\infty}^{\infty}\frac{d^3k}{(2\delta)^3}\,\tilde{G}_{km}(\omega,\boldsymbol{k})\,e^{-i\omega t + i\boldsymbol{k}x}$$

$$\delta(t)\delta^3(x) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{-\infty}^{\infty}\frac{d^3k}{(2\delta)^3}\,e^{-i\omega t + i\boldsymbol{k}x} \quad (10)$$

Substituting in we get:

$$\left(\Gamma_{ik} - i\omega D_{ik} - \rho V^2\delta_{ik}\right)\tilde{G}_{km}(\omega,\boldsymbol{k}) = \delta_{im}$$

$$\Rightarrow \tilde{G}_{km}(\omega,\boldsymbol{k}) = \left(\Gamma_{ik} - i\omega D_{ik} - \rho V^2\delta_{ik}\right)^{-1} \quad (11)$$

Let: $\left(\Gamma_{ik} - i\omega D_{ik}\right) = \Pi_{ik}$ and $\rho V^2 = \gamma$

Thus, we get:

$$\tilde{G}_{km}(\omega,\boldsymbol{k}) = \left(\Pi_{ik} - \gamma.I\right)^{-1} \quad (12)$$

$$\Pi_{ik} - \gamma.I = \begin{pmatrix} \Pi_{11}-\gamma & \Pi_{12} & \Pi_{13} \\ \Pi_{21} & \Pi_{22}-\gamma & \Pi_{23} \\ \Pi_{31} & \Pi_{32} & \Pi_{33}-\gamma \end{pmatrix} \quad (13)$$

$$\det\left[\Pi_{ik} - \gamma.I\right] = -\gamma^3 + \gamma^2(\Pi_{11}+\Pi_{22}+\Pi_{33})$$
$$-\gamma(\Pi_{11}\Pi_{22} + \Pi_{33}\Pi_{11} + \Pi_{22}\Pi_{33} - \Pi_{12}^2 - \Pi_{13}^2 - \Pi_{23}^2)$$
$$+\left(\Pi_{11}\Pi_{22}\Pi_{33} + 2\Pi_{12}\Pi_{23}\Pi_{31} - \Pi_{12}^2\Pi_{33} - \Pi_{13}^2\Pi_{22} - \Pi_{23}^2\Pi_{11}\right) = 0 \quad (14)$$

Let the Roots or Eigen values of $\Pi_{ik} - \gamma.I$ be γ_1, γ_2 & γ_3

And Phase Velocity Corresponding to the Eigenvalue γ_a is: $V_a = \sqrt{\frac{\gamma_a}{\rho}}$

In terms of the Eigenvalues/Roots the Determinant can be also written as:

$$\Delta(\gamma) = (\gamma-\gamma_1)(\gamma-\gamma_2)(\gamma-\gamma_3) \quad (15)$$

Thus,

$$(\Pi_{ik} - \gamma.I)^{-1} = \frac{1}{\Delta(\gamma)}\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \frac{\tilde{A}(\gamma)}{\Delta(\gamma)} \quad (16)$$

Where $\tilde{A} = (a_{ik})$ is the matrix of the cofactors of the expanded Matrix .

The Cofactors are:

$$A_{11}(\gamma) = \gamma^2 - \gamma(\Pi_{22}+\Pi_{33}) + \Pi_{22}\Pi_{33} - \Pi^2_{23}$$
$$A_{22}(\gamma) = \gamma^2 - \gamma(\Pi_{11}+\Pi_{33}) + \Pi_{11}\Pi_{33} - \Pi^2_{13}$$
$$A_{33}(\gamma) = \gamma^2 - \gamma(\Pi_{22}+\Pi_{11}) + \Pi_{22}\Pi_{11} - \Pi^2_{12}$$
$$A_{12}(\gamma) = \Pi_{12}\gamma + \Pi_{13}\Pi_{32} - \Pi_{12}\Pi_{33}$$
$$A_{13}(\gamma) = \Pi_{13}\gamma + \Pi_{12}\Pi_{23} - \Pi_{13}\Pi_{22}$$
$$A_{23}(\gamma) = \Pi_{23}\gamma + \Pi_{12}\Pi_{13} - \Pi_{23}\Pi_{11} \quad (17)$$

The components a_{ik} are derived by a cyclic permutation of the indices $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$. Now, we have to represent in the form of a sum of simple fractions with denominator exponents $(\gamma - \gamma_a)^p$. Here 'p' must not exceed the multiplicity of the respective root γ_a of.

Solving for the Green's function in K-space:

Representing Equation in the form of a sum of simple fractions. Here three cases may arise depending on the multiplicity of the roots.

Case I: 3 different roots

$$\frac{f(\gamma)}{\Delta(\gamma)} = \frac{a}{\gamma_1 - \gamma} + \frac{b}{\gamma_2 - \gamma} + \frac{c}{\gamma_3 - \gamma} \quad (18)$$

By the method of partial fractions, we obtain:

$$(\Pi_{ik} - \gamma.I)^{-1} = \frac{\tilde{A}(\gamma_1)(\gamma_2-\gamma_3)}{Q(\gamma_1-\gamma)} + \frac{\tilde{A}(\gamma_2)(\gamma_3-\gamma_1)}{Q(\gamma_2-\gamma)} + \frac{\tilde{A}(\gamma_3)(\gamma_1-\gamma_2)}{Q(\gamma_3-\gamma)} \quad (19)$$

Where, $Q = \gamma_1\gamma_2(\gamma_1-\gamma_2) + \gamma_2\gamma_3(\gamma_2-\gamma_3) + \gamma_3\gamma_1(\gamma_3-\gamma_1)$

So, the Green's Function for this Case is:

$$\tilde{G}(\omega,\boldsymbol{k}) = \frac{\tilde{A}(\gamma_1)(\gamma_2-\gamma_3)}{Q(\gamma_1-\gamma)} + \frac{\tilde{A}(\gamma_2)(\gamma_3-\gamma_1)}{Q(\gamma_2-\gamma)} + \frac{\tilde{A}(\gamma_3)(\gamma_1-\gamma_2)}{Q(\gamma_3-\gamma)} \quad (20)$$

Case II: 2 common roots

$$\frac{f(\gamma)}{\Delta(\gamma)} = \frac{a}{\gamma_1 - \gamma} + \frac{b}{\gamma_2 - \gamma} + \frac{c}{(\gamma_2 - \gamma)^2} \quad (21)$$

By method of partial fractions, we get:

$$\left(\Pi_{ik} - \gamma.I\right)^{-1} = \frac{\tilde{A}(\gamma_1)}{(\gamma_1-\gamma_2)^2}\left(\frac{1}{(\gamma_1-\gamma)}\right) + \left(1 - \frac{\tilde{A}(\gamma_1)}{(\gamma_1-\gamma_2)^2}\right)\left(\frac{1}{(\gamma_2-\gamma)}\right) + \frac{\tilde{A}(\gamma_2)}{(\gamma_1-\gamma_2)}\left(\frac{1}{(\gamma_2-\gamma)^2}\right) \quad (22)$$

Thus, the Greens function for this case is:

$$\tilde{G}(\omega,\boldsymbol{k}) = \frac{\tilde{A}(\gamma_1)}{(\gamma_1-\gamma_2)^2}\left(\frac{1}{(\gamma_1-\gamma)}\right) + \left(1 - \frac{\tilde{A}(\gamma_1)}{(\gamma_1-\gamma_2)^2} + \frac{\tilde{A}(\gamma_2)}{(\gamma_1-\gamma_2)}\left(-\frac{\partial}{\partial\gamma_2}\right)\right)\left(\frac{1}{(\gamma_2-\gamma)}\right) \quad (23)$$

This is the most common case, even the isotropic medium falls in this category.

Case III: 3 common roots

$$\frac{f(\gamma)}{\Delta(\gamma)} = \frac{a}{\gamma_1 - \gamma} + \frac{b}{(\gamma_1 - \gamma)^2} + \frac{c}{(\gamma_1 - \gamma)^3} \quad (24)$$

By method of partial fractions, we get the Greens function:

$$\tilde{G}(\omega,\boldsymbol{k}) = (\Pi_{ik} - \gamma.I)^{-1} = \frac{\hat{I}}{\gamma_1 - \gamma} - \frac{\tilde{A}'(\gamma_1)}{(\gamma_1-\gamma)^2} + \frac{\tilde{A}(\gamma_1)}{(\gamma_1-\gamma)^3} \quad (25)$$

This is for theoretical interest only and has physical meaning only in locally small physical volumes.

Results

The fluid inside the fractured medium can be represented by a 4th rank Viscosity tensor which can be written as:

$$\eta = \begin{bmatrix} \eta_1 + 2\eta_2 & \eta_1 & \eta_1 & 0 & 0 & 0 \\ \eta_1 & \eta_1 + 2\eta_2 & \eta_1 & 0 & 0 & 0 \\ \eta_1 & \eta_1 & \eta_1 + 2\eta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \eta_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \eta_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \eta_2 \end{bmatrix} \quad (26)$$

Where η_1 and η_2 is the Bulk and Shear viscosity respectively.

The Stiffness Tensor C of the effective fractured HTI medium without attenuation [5]:

$$C=\begin{bmatrix} (\lambda+2\mu)(1-\Delta_N) & (1-\Delta_N)\lambda & (1-\Delta_N)\lambda & 0 & 0 & 0 \\ (1-\Delta_N)\lambda & (\lambda+2\mu)\left\{1-\frac{(\Delta_N)\lambda^2}{(\lambda+2\mu)^2}\right\} & \lambda\left\{1-\frac{(\Delta_N)\lambda}{(\lambda+2\mu)}\right\} & 0 & 0 & 0 \\ (1-\Delta_N)\lambda & \lambda\left\{1-\frac{(\Delta_N)\lambda}{(\lambda+2\mu)}\right\} & (\lambda+2\mu)\left\{1-\frac{(\Delta_N)\lambda^2}{(\lambda+2\mu)^2}\right\} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-\Delta_T)\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & (1-\Delta_T)\mu \end{bmatrix} \quad (27$$

Here Δ_N and Δ_T is known as the normal and tangential weakness ($0<\Delta_N<1$, $0<\Delta_T<1$) and both are dimensionless [6]. The constants λ and μ are the Lamé constants of the Host rock.

For the dissipative HTI media we use the Complex Stiffness Tensor *C*, obtained from the Real stiffness Tensor C by substituting the real weaknesses Δ_N and Δ_T with complex weaknesses Δ_N and Δ_T:

$$\Delta_N \rightarrow \tilde{\Delta}_N = \Delta_N - i\Delta_N{}^I \qquad \Delta_T \rightarrow \tilde{\Delta}_T = \Delta_T - i\Delta_T{}^I \quad (28)$$

Thus, we get the Complex Stiffness Tensor as:

$$\tilde{C}=\begin{bmatrix} (\lambda+2\mu)(1-\Delta_N+i\Delta_N{}^I) & (1-\Delta_N+i\Delta_N{}^I)\lambda & (1-\Delta_N+i\Delta_N{}^I)\lambda & 0 & 0 & 0 \\ (1-\Delta_N+i\Delta_N{}^I)\lambda & (\lambda+2\mu)\left\{1-\frac{(\Delta_N-i\Delta_N{}^I)\lambda^2}{(\lambda+2\mu)^2}\right\} & \lambda\left\{1-\frac{(\Delta_N-i\Delta_N{}^I)\lambda}{(\lambda+2\mu)}\right\} & 0 & 0 & 0 \\ (1-\Delta_N+i\Delta_N{}^I)\lambda & \lambda\left\{1-\frac{(\Delta_N-i\Delta_N{}^I)\lambda}{(\lambda+2\mu)}\right\} & (\lambda+2\mu)\left\{1-\frac{(\Delta_N-i\Delta_N{}^I)\lambda^2}{(\lambda+2\mu)^2}\right\} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-\Delta_T+i\Delta_T{}^I)\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & (1-\Delta_T+i\Delta_T{}^I)\mu \end{bmatrix} \quad (29)$$

Substituting and in and after solving we get the complex eigenvalues that corresponds to the velocities as:

$$\begin{bmatrix} (1-\Delta_T+i\Delta_T{}^I)\mu - i\eta_2\omega \\ (1-\Delta_T+i\Delta_T{}^I)\mu - i\eta_2\omega \\ (1-\Delta_N+i\Delta_N{}^I)(\lambda+2\mu) - i(\eta_1+2\eta_2)\omega \end{bmatrix} \quad (30)$$

Thus, we have 2 roots common and 1 root different. This corresponds to the 2nd case and the Green's function can be represented by .

Substituting the Eigenvalues in we get the Green's function in Fourier Space as:

$$G(\omega,k)=\begin{bmatrix} G_1 & G_2 & G_2 \\ G_2 & G_2 & G_2 \\ G_2 & G_2 & G_2 \end{bmatrix} \quad (31)$$

Where:

$$G_1 = -\frac{k^2}{\rho\omega^2 + k^2\left\{(\lambda+2\mu)(1-\Delta_N+i\Delta_N{}^I) + 2(1-\Delta_T+i\Delta_T{}^I)\mu + i(\eta_1+2\eta_2)\omega\right\}}$$

$$G_2 = -\frac{k^2}{\rho\omega^2 + k^2\left\{(-1+\Delta_T-i\Delta_T{}^I)\mu + i\eta_2\omega\right\}}$$

Now we apply the Inverse Fourier Transform to get the Green's function in (x,t) space.

The Green's function in (x,t) space

In the previous section we have obtained the Green's function in the Fourier space. To obtain it in the (x,t) space, an Inverse Fourier transform have to be performed as follows:

$$G(t,x) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i\omega t + i\vec{k}\vec{x}} \tilde{G}(\omega,k)$$
$$\Rightarrow G(t,x) = \int_0^{\infty} dk k^2 e^{ik(\vec{n}.\vec{x})} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{-i\omega t} \tilde{G}(\omega,k) \quad (32)$$

Each of the terms in the above matrix have to be integrated separately and since there are only two unique terms, it saves us a lot of work as we only have to do two integrations. Since there are complex variables involved, a contour integration needs to be done properly choosing the poles that are inside the contour. Once the poles that satisfy the required conditions are selected we find the residues around them using *Cauchy's Residue Theorem.*

For both the unique terms while integrating over "ω" the contour is taken as in an anti-clockwise direction (Figure 2). Now the *principle of causality* (effects cannot precede the cause) is used to select the poles of interest. In the term $e^{-i\omega t}$, ($-i\omega t$) always must be positive to maintain the anti-clockwise direction of the contour. Thus, for ($t>0$), ω must be less than 0 for ($-i\omega t$) >0. Similarly, for ($t<0$), ω must be greater than 0 for (($-i\omega t$))>0. By the principle of causality, we reject the poles in the ($t<0$) region thus we calculate the residue for the poles only in the lower half where $\omega<0$.

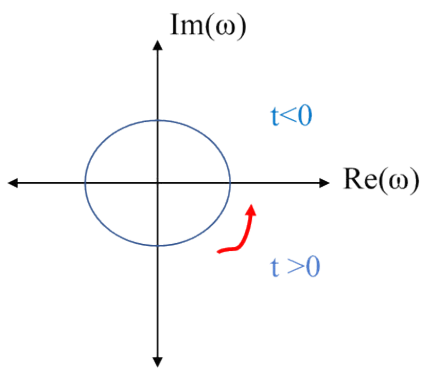

Figure 2: The contour for the integration over ω.

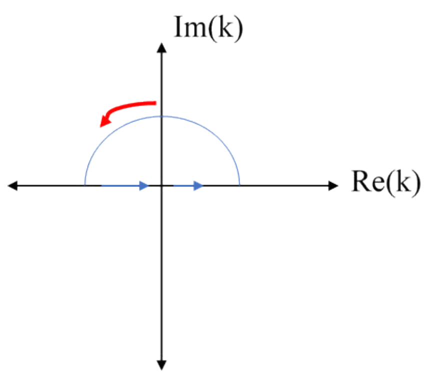

Figure 3: The contour for the integration over k.

Similarly, while integrating over "k" the contour is taken as in an anti-clockwise direction (Figure 3). As the integration limits are from 0 to O the contour is taken only in the upper half and the poles in the lower half of the plane are ignored.

So finally after the integration of the two unique terms we obtain the greens' function in (x,t) space as:

$$G(t,x)=\begin{bmatrix} A_1e^{-\kappa_1 x+\phi_1 t} & A_2e^{-\kappa_2 x+\phi_2 t} & A_2e^{-\kappa_2 x+\phi_2 t} \\ A_2e^{-\kappa_2 x+\phi_2 t} & A_2e^{-\kappa_2 x+\phi_2 t} & A_2e^{-\kappa_2 x+\phi_2 t} \\ A_2e^{-\kappa_2 x+\phi_2 t} & A_2e^{-\kappa_2 x+\phi_2 t} & A_2e^{-\kappa_2 x+\phi_2 t} \end{bmatrix} \quad (33)$$

Where:

$$A_1=-\frac{(55.8309)(-1+\Delta_N-i\Delta_N{}^I)(\lambda+2\mu)\rho^3}{(\eta_1+2\eta_2)\{-(-1+\Delta_N-i\Delta_N{}^I)(\lambda+2\mu)\rho\}^{1/2}}$$

$$\kappa_1=2\left\{\frac{(-1+\Delta_N-i\Delta_N{}^I)(\lambda+2\mu)\rho}{(\eta_1+2\eta_2)^2}\right\}^{1/2}$$

$$\phi_1=2i\left\{\frac{(-1+\Delta_N-i\Delta_N{}^I)(\lambda+2\mu)}{(\eta_1+2\eta_2)}\right\}$$

$$A_2=-\frac{(17.7715)(-1+\Delta_T-i\Delta_T{}^I)\mu\pi\rho^2}{\eta_2\{-(-1+\Delta_T-i\Delta_T{}^I)\mu\rho\}^{1/2}}$$

$$\kappa_2=2\left\{\frac{(-1+\Delta_T-i\Delta_T{}^I)\mu\rho}{\eta_2{}^2}\right\}^{1/2}$$

$$\phi_2=2i\frac{(-1+\Delta_T-i\Delta_T{}^I)\mu}{\eta_2}$$

Conclusion

So, the Green's function is finally derived. This Green's function can subsequently be used to interpret many wave scattering, reflection, refraction, and propagation effects where viscoelastic layers are present. Thus, subsequently it can be used for determination of hydrocarbon layers, as hydrocarbons have viscoelastic properties. Further works on this topic will follow.

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