



Micromagnetic Theory of Time-Dependent Suprastriction

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Abstract

A micro magnetic theory is described for the treatment of suprastriction in a disc of a type-II superconductor with a time-oscillating external magnetic field perpendicular to the disc plane. By the action of the field vortices are driven out of the disc and reappear in the disc in a periodic manner, leading to a time and position-dependent magnetization $M(r, t)$. This magnetization is calculated by the time-dependent Ginzburg-Landau equation of motion for the vortices. Thereby effects of the pinning of vortices at defects or of the intrinsic pinning of vortices in high- T_c superconductors with very short correlation lengths can be included. The magneto elastic interaction between the magnetization $M(r, t)$ and the lattice generates magnetostrictive strains, which in the case of superconductors are called suprastrictive strains. They are calculated in the present paper.

Keywords: Superconductors; Magnetosriction; Static magnetic; Magnetostrictive; Quasiplastic

Introduction

Magnetostriction of superconductors is called suprastriction. A review on suprastriction is given in [1]. There the results of suprastriction studies of different superconducting compounds made in former years are presented. A phenomenological description of suprastriction induced by magnetic flux pinning is given. A micro magnetic theory of suprastriction is given in [2]. There a subdivision of the suprastriction into spontaneous suprastriction, forced suprastriction (generated by the application of a magnetic field, in [1] called induced suprastriction) and form effect is given. In [3] analytical solutions of the Ginsburg-Landau equation for deformable superconductors in a weak static external magnetic field are given [4] reports on the flux-pinning induced magnetostriction in hard type-II superconductors, investigated by magneto elastic equations. In [5] exact analytical results are obtained for the flux-pinning-induced magnetostriction in cylindrical type-II superconductors placed in an external static magnetic field parallel to the cylinder axis.

In all these former papers static situations have been considered. In the present paper a micro magnetic theory of suprastriction is developed in a thin cylindrical disc of a type-II superconductor with a

time-oscillating magnetic field perpendicular to the disc plane. By this field vortices are driven out of the sample and reappear in the sample in a periodic manner. This generates a time and position-dependent magnetization $M(r, t)$ which is calculated by the time-dependent Ginzburg-Landau equation.

$$\alpha\psi + \beta/\psi^2 + 1/(2m^*)(-i\hbar\nabla + 2e\mathbf{A})\psi = 0 \quad (1)$$

$$j = -2e/m^* \text{Re}(\psi(-i\hbar\nabla + 2e\mathbf{A})\psi) \quad (2)$$

Here α and β are phenomenological parameters, m^* is the effective mass of the Cooper pairs, ψ is the complex order parameter, $-2e$ is the charge of a Cooper pair, \mathbf{A} is the vector potential which is related to the magnetic induction \mathbf{B} by $\mathbf{B} = \text{curl}(\mathbf{A})$.

Pinning forces exerted on the vortices can also be included in the theory. There are many experiments on time-dependent suprastriction, concerning the behavior of flux lines the type-II superconductors. A physical understanding of the processes therefore is very important, and it is the scope of the present papers to give these physical explanations.

Literature Review

Micromagnetic theory of time-dependent suprastriction

The theory of the present paper is based on a micromagnetic description of magnetostriction in magnetic materials, outlined in the book of [6]. It is an extension of the static suprastriction theory of dynamic situations.

The magnetostrictive strain is generated by the magnetoelastic interaction of the magnetization $M(r, t)$ with the lattice. The magnetization enters the theory *via* the quasiplastic strain tensor $\epsilon_{ik}^Q/7$ with the components.

$$\epsilon_{ik}^Q = 3/2 \Lambda_{100} (\gamma_i^2 - 1/3) \text{ for } i=k=1,2,3. \quad (3)$$

$$\epsilon_{ik}^Q = 3/2 \Lambda_{111} \gamma_i \gamma_k \text{ for } i \neq k$$

The quantities Λ_{100} and Λ_{111} are magnetostriction constants for cubic crystals, which correspond to the fractional change of the sample length upon saturation in $\langle 100 \rangle$ -direction and upon saturation in $\langle 111 \rangle$ direction, respectively. The $\gamma_i = \gamma_i(r, t)$ are the direction cosines of $M(r, t)$.

The elastic strain tensor of the sample generated by the magneto elastic interaction of the magnetization $M(r, t)$ with the lattice, i.e., the magnetostrictive strain tensor (in superconductors called suprastrictive strain tensor) ϵ_{el} , is calculated by minimizing the magnetoelastic potential $\phi_{el}/6$.

$$\phi_{el} = 1/2 \int \int \int (\epsilon_{ij} Q_{ijkl} - \epsilon_{ij} C_{ijkl} - \epsilon_{ij} \epsilon_{kl} - (\epsilon_{def} + \epsilon_{ext}) C_{ijkl} (\epsilon_{ij} + \epsilon_{kl})) d^3r \quad (4)$$

With respect to $\epsilon_{el}(r, t)$. Here the symbol $\bullet\bullet$ denotes the tensor product, C is the tensor of elastic constants, and the tensor ϵ_{def} describes the strain in systems in which the vortices are pinned by defects. The tensor ϵ_{ext} describes the strain due to external forces. In the present paper external forces are not considered [7].

In a type-II superconductor the magnetization is generated by the magnetic flux lines. When applying a time-oscillating external

magnetic field perpendicular to the disc plane, then vortices are driven out of the disc and reappear in the disc in a periodic manner. An example for a highly-resolved picture of the flux distribution at various fields is given in the figure. There are no figures for time-dependent suprastriction, therefore I present a figure for non-time-dependent suprastriction in the type-I superconductor Pb (Figure 1).

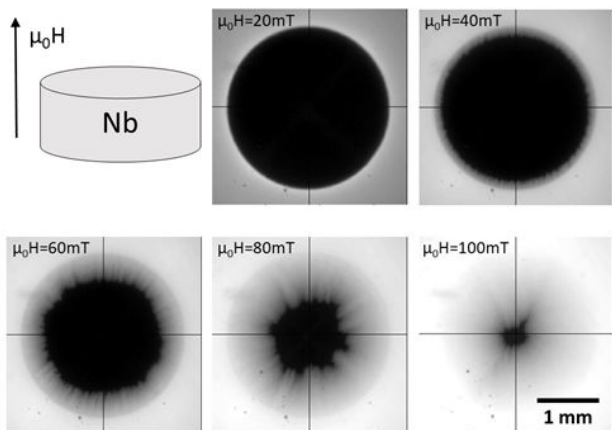


Figure 1: A high-resolution picture of the flux distribution for various magnetic field H for the type-I superconductor Pb. The figure is given to me by Joachim Albrecht from the Hochschule Aalen, Germany.

This process can be described by the time-dependent Ginzburg-Landau equation, see equations 1 and 2 and for instance [8]. Thereby pinning forces exerted on the vortices by defects can be implemented, which of course depend on the types and the densities of the defects. In high- T_c superconductors with very small coherence lengths there is also an intrinsic pinning [9, 10] related to the positions of the vortices in the atomic lattice. When the Ginzburg-Landau equation is atomically discretized, then intrinsic pinning effects can be investigated. Altogether, simply speaking, the pinning forces are transmitted to the sample, thereby changing its size.

In the present theory the momentary magnetization enters the quasiplastic strain tensor, and thus the suprastriction is determined by the momentary magnetization. In reality, however, the solid requires some time to react on a change of the magnetization, i.e., the suprastriction does not instantaneously follow the change of the magnetization. Physically, the change of the extensions of the system by magnetostriction is performed by a change of the phononic occupation numbers. Therefore the present theory is precise only for slow variations of the magnetization in time, i.e., for slowly varying time-oscillations of the external magnetic field [9].

For faster dynamics the theory can be extended by an ansatz for the realistic quasiplastic strain tensor $\mathcal{E}Q_{real}$. We start with the quasiplastic strain tensor for a static situation, $\mathcal{E}Q_{static}$. When the magnetization dynamics starts, then there is an additional contribution $\mathcal{E}Q_{dyn}$ the realistic tensor.

$\mathcal{E}Q_{real}$ is calculated by the ansatz

$$\mathcal{E}Q_{real} = \mathcal{E}Q_{static} + \mathcal{E}Q_{dyn} (1 - \exp(-\tau/\tau_c)) \quad (5)$$

Here τ is the characteristic time for the change of the magnetization in time, and τ_c is the characteristic time which the solid needs to react on the change of the magnetization, typically a relaxation time for the phononic occupation numbers. These relaxation times are typically of the order of a few ps to a few hundred ps [11]. For relatively slow variations of the magnetization used in experiments on the time-dependent suprastriction these effects of the finite relaxation time of the phononic occupation numbers are not very relevant. For very fast changes of the magnetization they are relevant. These effects have been never considered in former papers on time-dependent changes of the magnetization and the magnetostriction, and it is a further scope of the present manuscript to outline this important point [10].

Results

Transfer of angular momentum from the magnetic vortices to the atomic lattice

The vortices carry magnetic moments. Magnetic moments are related to angular momenta. When the vortices are driven out of the disc and reappear in the disc in a periodic manner due to the action of the time-oscillating external magnetic field, then the angular momenta are transferred at least in part to the atomic lattice. A precondition for the angular momentum conservation is the rotational invariance of the system. This is not fulfilled in the experiment on the superconducting disc which is not rotationally invariant. Therefore the angular momentum of the flux lines is not necessarily completely transferred to the lattice. Nevertheless, it is interesting to know how much of it is transferred. As discussed in section II, the magnetoelastic interaction of the time-dependent magnetization $M(r, t)$ generates elastic strains $\mathcal{E}_{el}(r, t)$ in the disc. As taught in textbooks on elasticity theory, the angular momentum due to these time-dependent strains carry an angular momentum of size.

$$L = \iiint \mathcal{E}_{,ijk} \delta_{jm} \sigma_{mk} d^3r \quad (6)$$

Here σ_{mk} are the components of the strain tensor which may be calculated from the components $\mathcal{E}_{el, np}$ via Hooke's law. The \mathcal{E}_{ijk} are the components of the Levi-Civita tensor, and δ_{im} is the Kronecker delta.

There is a possibility to measure the transferred angular momentum in the following way. When the disc is not rigidly fixed on a sample holder, but hangs at a thread, so that it can rotate freely, then its rotation carries the angular momentum calculated by equation (6). By this it is possible to check experimentally the theoretical result for the transferred angular momentum. Because of the inertia of the disc this should be done for rather small frequencies of the time-oscillating external magnetic field.

Discussion

In the present paper former investigations of suprastriction in type-II superconductors, especially the micromagnetic theory of suprastriction by Kronmüller [2], have been extended from time-independent situations to time-dependent situations generated by an external time-oscillating magnetic field. A thin disc of a type-II superconductor with an external field perpendicular to the disc plane is considered. By the time-oscillating magnetic field vortices are driven

out of the disc and reappear in the disc in a periodic manner. This generates a time and position-dependent magnetization $M(r, t)$ of the disc. This magnetization can be calculated by the time-dependent Ginzburg-Landau theory. Thereby forces on the vortices due to pinning at defects or due to intrinsic pinning effects in high- T_c superconductors with very small correlation lengths can be included.

Conclusion

Based on a micro magnetic approach the elastic (suprastrictive) strains can be calculated. The theoretical results may lead to experimental investigations of the time-dependent suprastriction, yielding possibly very interesting new phenomena. As discussed in section III it is also possible to check the theoretical results for the transfer of angular momentum from the system of vortices to the atomic lattice. It is not the scope of the present to perform numerical calculations for specific superconductors. Readers which are interested in such numerical results can insert in the equations the specific material parameters for the considered superconductor (see the lines below equations 1 and 2) to get numerical results. It was the scope of the present paper to develop a theory which explains the physics behind the time-dependent suprastriction and to provide the equations which can be used to get numerical results readers of the paper interested in this can use my equations.

References

1. Seeger A (1970) Superconductivity and physical metallurgy. Metall Trans 1:2987-2996.
2. Bardeen J (1978) Motion of flux lines in nearly pure superconductors. Phys Rev B 17: 147-152.
3. Brandt EH (1977) Elastic response of the fluxoid lattice with planar pinning forces. Physica Status Solidi 84: 269-281.
4. Brandt EH (1977) Instability of the flux-line lattice in a type-II superconductor containing planar imperfections. Physica Status Solidi 84: 637-647.
5. Brandt EH, Essmann U (1975) Spatial fluctuations of the order parameter and the peak effect in type-II superconductors. Phys Lett A 51: 45-46.
6. Kammerer U (1969) Wechselwirkungskräfte zwischen einer Flußlinie und einer parallelen Versetzung. Physica Status Solidi 34: 81-94.
7. Schmucker VR (1977) Das flußdichteprofil zylindrischer supraleiter zweiter art. Philos Mag 35: 453-469.
8. Schneider E, Kronmüller H (1976) The elementary interaction between a crystal dislocation and the flux line lattice of a type II superconductor. Physica Status Solidi 74: 261-273.
9. Fischer K, Teichler H (1976) On the correlation between flux line lattice and crystal lattice. Phys Lett A 58: 402-404.
10. Obst B, Brandt EH (1978) Phase transition from the triangular to the square flux line lattice in type II superconductors with small κ . Phys Lett A 64: 460-462.