



Research Article

# Synchronizing Two Frequently-Cited High-Capacity Image Hiding Methods (Modulus-Based Vs. LSB-Based)

Ja-Chen Lin<sup>1\*</sup>

## Abstract

This study briefly reviews and then connects a Modulus-based image hiding method published in 2003 and an LSB-based (Least-Significant-Bits) image hiding method published in 2004. Both methods are frequently cited; and the reason is because of their high hiding capacity (the size of the data can be hidden in an image) and low distortion to the host image; along with the benefit of being simple. This study synchronizes the two methods as one simple method so that the PSNR prediction formula of the 2003 method can also be used by the 2004 method to predict the PSNR of the stego image. The gaps between the two methods are thus connected. Two examples are included to tell the readers how to use the simple method to hide data in different situations (on-line vs. off-line). Furthermore, a new theorem is also developed to predict the Mean "Absolute" Error (MAE) of the stego image as  $0.25m$  (or  $0.25m - 0.25/m$  if  $m$  is odd) when the simple method uses base  $m$  in the hiding system to get a hiding rate being  $(\log m)/\log 2$  bits per pixel (bpp). Experimental results show that the predicted MAE is very close to the actual MAE value. The capacity-PSNR performances are also compared with that of several methods published recently; and it is observed that this 2003-2004 simple method is still very competitive nowadays. Therefore, we can say that the method is simple, with low-distortion; of high hiding capacity, and equipped with simple formulas to predict excellently both MAE and PSNR of the stego images in a very easy to compute manner.

## Keywords

Data hiding; Least Significant Bits (LSB) based; Modulus based; PSNR estimation formula; Mean Absolute Error (MAE); Estimation formula

## Introduction

Data hiding has been a popular research topic for decades [1-10]. In data hiding, people try to hide secret data in some cover media. High hiding capacity and low distortion to the cover media are usually required when a method is designed. Both the Modulus-based hiding method [1] and the LSB-based (Least-Significant-Bits) hiding method [2] are of high hiding capacity and low distortion; so these two methods are frequently cited. The study here synchronizes these two methods as one simple method; therefore, the users of the LSB method [2] can also use all mathematical formulas listed [1];

especially, the PSNR-prediction formula [1]. Moreover, we will also design here a new formula which is useful in predicting the Mean Absolute Error (MAE) of the stego images. Of course, this new formula can also be applied to both methods [1,2].

Notably, so far, most of the research papers in image hiding field show PSNR values (computed from Mean Square Error (MSE) values), rather than MAE values, of the stego images. Willmott and Matsuura [11] suggest the use of MAE to replace MSE or square root of MSE (the RMSE); whereas some [12] still suggests the favor of RMSE. In certain applications or in other fields, some researchers begin to provide MAE values or list both MAE and PSNR simultaneously in the same paper; for example, the audio hiding system [13], or the computer-aided breast tumor segmentation system [14]. Therefore, we try to list both MAE and PSNR in our report here so that, in the future, the researchers can use these MAE and PSNR values to compare against their own methods' MAE and PSNR.

The PSNR-bpp performance of some other high-capacity and low-distortion methods [4-8] recently appeared in literature are also compared against that of the simple methods [1,2]. The methods being compared include the multi-layer embedding method of Tang, Hu et al. [5]; the tunable method of Kanan and Nazeri [4] who use genetic algorithm to get nearly optimal search; the adaptive method designed by Maleki et al. [6], together with the other adaptive method of Sadashiv and Rao [8]. We will show that the simple methods [1,2] are still very competitive nowadays, although they were originally designed in 2003-2004.

## Materials and Methods

### Summarizing Thien and Lin's Module-based hiding method

Consider hiding a sequence of digits of base- $m$  (each digit is in the range  $\{0,1,2,3,4,5,\dots,m-2,m-1\}$ ); then embed each digit  $x$  in a pixel  $y$  of a given host image (one digit per pixel). Due to hiding, original pixel value  $y$  is changed to the stego pixel value  $y'$ . It is easy to prove that the embedding formula used by Thien and Lin [1] can be simplified as

$$y' = \text{round}\left(\frac{y-x}{m}\right) \times m + x \quad (1)$$

where the *round* operator rounds its argument to the nearest integer. The extraction of the hidden digit  $x$  from stego pixel  $y'$  is also very simple, just use

$$x = (y') \bmod m \quad (2)$$

**Remark 1:** Of course, if the new pixel value  $y'$  is less than 0 (or larger than 255), do the adjustment by using  $y'+m$  (or  $y'-m$ ) as the final pixel value. Notably, this  $\pm m$  adjustment will not affect the value  $x$  extracted by (2) due to  $\bmod_m$  operation.

A particularly important quantity property provided by Thien and Lin [1] is:

**Property 3 of Thien and Lin's Module-based hiding method:** If the embedded data  $\{x\}$  is uniformly distributed, then after hiding in each pixel a message digit, the expected PSNR of the whole stego image is approximately

\*Corresponding author: Ja-Chen Lin, Professor, Department of Computer and Information Science, National Chiao Tung University, Hsinchu, Taiwan 300, Taiwan, Fax: +886-3-5721490; E-mail: jclin@cis.nctu.edu.tw

Received: December 31, 2015 Accepted: April 05, 2016 Published: April 11, 2016

$$\text{PSNR}=10\text{Log}\{12(255^2)/(m^2-1)\} \quad \text{when } m \text{ is odd}; \quad (3)$$

$$\text{PSNR}=10\text{Log}\{12(255^2)/(m^2+2)\} \quad \text{when } m \text{ is even}. \quad (4)$$

**End of Property 3 of Thien and Lin’s Module-based hiding method:** Notably, by Equation (4), the PSNR of the stego image is

$$10\text{Log}\{12(255^2)/(m^2+2)\} = 51.1411; 46.3699; 40.7272; \text{ and } 34.8064 \quad (5)$$

for hiding 1 bit per pixel (bpp), hiding 2 bpp, hiding 3 bpp, and hiding 4 bpp, respectively; because we only need to plug in Eq. (4) the parameter value

$$m=2^k = 2, 4, 8, 16, \quad (6)$$

respectively. These PSNR values are extremely close to the experimental values obtained in Tables 1 and 2 of Chan CK et al.’s research [2], the experimental values they obtained were {51.1410, 46.3699, 40.7271, 34.8062} for host image Lena, {51.1414, 46.3691, 40.7253, 34.8021} for host image Baboon, {51.1405, 46.3700, 40.7273, 34.8065} for host image Jet, {51.1410, 46.3702, 40.7270, 34.8060} for image Scene.)

**Summarizing LSB-based hiding method**

Motivated by their own graceful observation [3], Chan and Cheng [2] proposed a method in which each host pixel hides a *k*-bits binary-value *x*. The method can also be summarized as Eq. (1) and (2); except that only uses

$$m=2^k \in \{2,4,8,16\}.$$

Below we explain why the hiding method in [2] can also be described as Eq. (1) and (2), using  $m=2^k$ . Notably, Eq. (5) of [2] also uses the same extraction operation  $x=(y')_{\text{mod}m} = (y')_{\text{mod}2^k}$  stated here in Eq. (2) to extract data digit *x*. So, we only need to explain why the stego pixel value *y'* [2] is identical to the one produced here by Eq. (1). Since also uses  $x=(y')_{\text{mod}m}$  to extract *x*, the stego pixel value *y'* of must satisfy  $y=lm+x$  for some integer *l*. In Chan CK, Cheng LM research [2], their design to choose the integer *l* is according to the merit that the

distance  $|y'-y|$  between the stego pixel value  $y'=lm+x$  and the original pixel value *y* should be as small as possible. The rounding function in Eq. (1) above chooses *l* as  $l = \text{round}\left(\frac{y-x}{m}\right)$ ; and this *l* can achieve this minimal distortion goal. To see the reason, let  $a = (y)_{\text{mod}m}$ , so the original pixel value *y* is  $y=nm+a$  for some nonnegative integer  $n = \lfloor y/m \rfloor$ . Here,  $\lfloor \cdot \rfloor$  is the round-down operator, also known as the floor function. The distortion caused by embedding is

$$|y'-y| = |(l-n)m+(x-a)|.$$

Traditional LSB method just let  $l=n$ , i.e. the integer portion of  $(y/m)$ ; and this implies  $|y'-y| = |(x-a)|$  which is at most *m*-1 because both *x* and *a* are in  $\{0,1,\dots,m-1\}$ . Chan and Cheng [2] choose *l* by considering not only *n*, but also other integers near *n*, i.e., *n*+1 and *n*-1. Since more possible candidates are considered for *l*, the resulting system improves the distortion of traditional LSB. As for Eq. (1) above, since

$$x \in \{0,1,\dots,m-1\} \text{ and } a \in \{0,1,\dots,m-1\} \Rightarrow -m < (a-x) < m,$$

we see that  $\text{round}((a-x)/m)$  is either -1 or 0 or 1. Hence, the *y'* of Eq. (1) is

$$\begin{aligned} y' &= \text{round}\left(\frac{y-x}{m}\right) \times m + x = \text{round}\left(\frac{m+a-x}{m}\right) \times m + x \\ &= (n + \text{round}\left(\frac{a-x}{m}\right)) \times m + x \end{aligned}$$

i.e. the *y'* of Eq. (1) is also an  $lm+x$  with possible values of *l* being *n*-1 or *n* or *n*+1. Finally, the rounding operator automatically adjusts the value of *l* to obtain minimal distortion between *y* and *y'*. For example, if  $a-x \geq 0.5m$ , then  $\text{round}((a-x)/m) = \text{round}(0.5\dots) = 1$ , then Eq. (1) becomes  $y'=(n+1)m+x$ , so  $y'-y = ((n+1)m+x) - (nm+a) = m-(a-x) = m-(m*(0.5\dots))$  becomes a number between 0 and 0.5*m*. The distortion is thus  $|y'-y| \leq 0.5m$ . This is better than the distortion  $|(nm+x)-y| = |(nm+x) - (nm+a)| = |x-a| \geq 0.5m$  caused by using  $l=n$ , and also better than the distortion  $|(n-1)m+x)-y| = |-m+x-a| = |m+a-x| \geq 1.5m$  caused by using  $l=n-1$ . Therefore, for the case

**Table 1:** The PSNR-bpp performance of the method here vs. the method [6].

bpp , i.e. number of hidden bits per pixel	PSNR of the simple method using Eq. (1-2)	PSNR of the adaptive method in Ref. [6]
1.58 bpp (base m=3)	49.89	
1.58 bpp		47.56
2.31		43.54
2.32 (base m=5)	45.12	
2.58 (base m=6)	43.12	
2.63		38.93
2.78		38.16
2.81 (base m=7)	42.11	
3.17 (base m=9)	39.89	
3.31		37.55
3.32 (base m=10)	38.84	
3.42		36.82
3.46 (base m=11)	38.13	
3.47		36.76
3.58 (base m=12)	37.28	
3.70 (base m=13)	36.67	
3.72		35.61
3.81 (base m=14)	35.96	
3.91 (base m=15)	35.42	
4.06		33.14
4.10 (base m=17)	34.33	

**Table 2:** MAE (Mean Absolute Error) comparison.

The method using Eq. (1)		Table 2 of Ref. [9]		Ref. [10]	
MAE	bpp	MAE (average of 4 MAEs)	bpp	MAE(average of 3 channels of color Lena)	bbp (hidden bits per host byte)
MAE = 0.5	1	1.59	0.99bpp	0.39	1
MAE=0.66...	1.58	2.05	1.48bpp	0.61	1.33
MAE= 1	2	2.48	2.06bpp	1.42	1.67
MAE=1.2	2.32			2.28	2
MAE=1.5	2.58			2.89	2.33
MAE = 1.71	2.81	2.80	2.75bpp		

**Table 3:** MAE (Mean Absolute Error) and PSNR: predicted values vs. experimental results.

$m$ ; bpp = $\log_2 m$	Predicted MAE, and predicted PSNR	The host image			
		Lena	Peppers	F16	Baboon
$m = 2$ ; bpp= 1	MAE = 0.25m=0.5 PSNR = 51.14	0.5002 51.1380	0.5001 51.1396	0.5011 51.1309	0.5009 51.1327
$m = 3$ ; bpp=1.58	MAE= $m/4-1/(4m)$ =0.6667 PSNR = 49.89	0.6635 49.9120	0.6707 49.8078	0.6659 49.8955	0.6687 49.8754
$m = 4$ ; bpp=2	MAE= 0.25m=1 PSNR = 46.37	1.0009 46.3655	1.0113 46.2672	0.9991 46.3767	1.0001 46.3704
$m=5$ bpp=2.32	MAE= $m/4-1/(4m)$ =1.2 PSNR = 45.12	1.2007 45.1210	1.2109 45.0106	1.1968 45.1344	1.2009 45.1086
$m = 6$ ; bpp=2.585	MAE= 0.25m=1.5 PSNR = 43.12	1.5014 43.1103	1.5129 43.0196	1.4964 43.1365	1.5055 43.1046
$m = 7$ ; bpp=2.81	MAE= $m/4-1/(4m)$ =1.7143 PSNR = 42.21	1.7108 42.1245	1.7279 42.0004	1.7231 42.0803	1.7168 42.1005
$m = 8$ ; bpp=3	MAE= 0.25m=2.0 PSNR = 40.73	2.0005 40.7271	2.0209 40.6015	1.9716 40.8158	2.0050 40.7093
$m = 9$ bpp=3.17	MAE= $m/4-1/(4m)$ =2.2222 PSNR = 39.89	2.2160 39.9107	2.2430 39.7575	2.2224 39.8849	2.2236 39.8816
$m=10$ ; bpp=3.32	MAE = 0.25m=2.5 PSNR = 38.84	2.5037 38.8292	2.5279 38.7042	2.5017 38.8265	2.5046 38.8213
$m = 11$ ; bpp=3.46	MAE= $m/4-1/(4m)$ =2.7273 PSNR=38.13	2.7182 38.1536	2.7611 37.9850	2.7302 38.1215	2.7266 38.1269
$m = 12$ ; bpp=3.585	MAE= 0.25m=3 PSNR = 37.28	2.9919 37.2955	3.0261 37.1555	2.9930 37.2878	2.9991 37.2846
$m=13$ ; bpp=3.70	MAE= $m/4-1/(4m)$ =3.2308 PSNR = 36.67	3.2317 36.6719	3.2630 36.5361	3.2289 36.6719	3.2279 36.6731
$m = 14$ ; bpp=3.81	MAE = 0.25m=3.5 PSNR = 35.96	3.5106 35.9334	3.5436 35.8047	3.5038 35.9573	3.5110 35.9355
$m = 15$ ; bpp=3.91	MAE= $m/4-1/(4m)$ =3.7333 PSNR = 35.42	3.7413 35.4086	3.7911 35.2530	3.7147 35.4500	3.7304 35.4229
$m=16$ ; bpp=4	MAE= 0.25m=4 PSNR = 34.81	3.9969 34.8119	4.0520 34.6505	4.0133 34.7627	3.9848 34.8303

$a-x \geq 0.5m$ , the  $y'$  created by Eq. (1) is identical to the best  $lm+x$  chosen by the method [2]. The case  $a-x \leq -0.5m$  can be proved likewise to get minimal distortion by using  $l=n-1$  which is also automatically determined by the rounding operator of Eq. (1). Finally, analogous prove also shows that the case  $-0.5m < a-x < -0.5m$  will get minimal distortion by using  $l=n$  which is also automatically determined by the rounding operator of Eq. (1). The tedious proof of these two cases is omitted to save paper length. Interested readers can see the proof of the Lemma later to get more idea of how to prove here for the case  $a-x \leq -0.5m$  or the case  $-0.5m < a-x < -0.5m$ .

As a final remark, Eq. (11) stated that the worst-case MSE of their method is a constant  $q$  times the worst-case MSE of the traditional

LSB method [2], with  $q$  being 1, 4/9, 16/49, and 64/225; when each pixel hides 1, 2, 3, and 4 bits; respectively.

### Comparing methods (Modulus-based vs. LSB-based)

The two methods proposed by Thein and Lin and Chan and Chang were designed individually [1,2]; and both are submitted in 2002, the work of Chan and Cheng [2] was submitted 80 days earlier, although the study by Thein and Lin [1] was published earlier due to the direct acceptance. Both Thein and Lin [1] provide some mathematical properties emphasizing the estimated PSNR when random data are embedded, whereas Chan and Chang [2] provide some mathematical properties related to worst-case error.

In a rough sense, both methods used similar approach (Eq. (1-2)), but LSB-based method [2] only used  $m=2, 4, 8, 16$  in Eq. (1-2), while Ref. [1] had no such restriction because it was motivated by modulus function directly. As a result, Ref. [1] can be used in wider applications; although the two methods are identical when dealing with a sequence of (1-bits, 2-bits, 3-bits, or 4-bits) binary data.

**Example 1:** One of the application examples of Thien CC, Lin JC [1] is, as mentioned if the hiding system is used on-line, and the incremental data that keep on coming in are decimal and of variable length (say, 87597257364, then 129, then 76887564923, then 9654988, then 45, and so on), then it is difficult to transform data to binary numbers for the on-line real-time hiding, because nothing can be done about the transform on receiving 8759725736 (at least not until the final digit 4 is received and the number is known to be 87597257364). Of course, the precise transform,  $(87597257364)_{10} = (1 \cdot \dots \cdot 10100)_2$ ;  $(129)_{10} = (1000001)_2$ ; etc., can be skipped and instead, the so-called real-time “digit-by-digit” transform (i.e.  $9 = (1001)_2$ ; ... ;  $0 = (0000)_2$ ), can be used to transform instantly the decimal number 8-7-5-9-7-2-5-7-3-6-4 into a binary sequence 1000-0111-...-0100. However, using 4-bit LSB substitution method of [2] (i.e. using  $m=2^4=16$  in Eq. (1-2)) to embed this instantly-transformed 4-bit binary sequence will result in worse PSNR, as compared with using  $m=10$  (as suggested by [1]) to hide original decimal sequence. This can be seen from Eq. (4) above.

**Example 2:** Even if binary-sequence secret are deal with, Thien and Lin’s method [1] can still get more benefits in many cases. For example, if  $3 \times 512 \times 512$  bits are to be hidden in an image of  $512 \times 512$  pixels, then, both methods [1,2] yield the same result (by using  $m=2^3=8$  in Eq. (1-2)). If  $4 \times 512 \times 512$  bits are to be hidden, then both methods [1,2] still yield identical result (by using  $m=2^4=16$  in Eq. (1-2)). However, if the number of secret bits are larger than  $3 \times 512 \times 512$  but smaller than  $4 \times 512 \times 512$ ; then, using Thien and Lin method [1] which can allow the users to choose an  $m \in \{9, 10, 11, 12, 13, 14, 15, 16\}$ , then use this  $m$  in Eq. (1-2), after transforming the secret into a string of base- $m$  digits. This is different from the bit-based method [2] which will use 4-bits version in this case, i.e. use  $m=2^4=16$  in Eq. (1-2), and thus cause larger distortion and yields smaller PSNR value of the stego image.

Likewise, if the secret data has more than  $4 \times 512 \times 512$  bits yet less than  $\log_2 m \times 512 \times 512$  bits for an  $m=17$  or  $m=18$ ; then using 5 bpp in the LSB-based method will cause error become totally unacceptable, whereas using base  $m$  in Eq. (1-2) might still has a small chance to succeed.

### Combining methods (Modulus-based and LSB-based)

To combine Methods Thien CC, Lin JC [1] and Chan CK, Cheng LM [2], just use Eq. (1-2) for embedding and extraction. As for the prediction of PSNR, there are two resources:

- 1) in the special case  $m=2^k$ , i.e.  $\{2, 4, 8, 16\}$ , the readers can refer to [2] to use the worst-case PSNR of the stego images.
- 2) On the other hand, no matter the integer  $m$  is  $2^k$  or not, Property 3 of [1], i.e. Eq. (3-4) above, usually gives a good estimate of the PSNR of the stego images.

As stated in the introduction section, more and more scientific or industrial fields begin to include MAE in their reports. Therefore, other than the PSNR prediction formula of Thien CC, Lin JC [1], here we also develop a new theorem to predict the Mean Absolute

Error (MAE) of the stego file created by Eq. (1). This MAE prediction formula can be applied directly to the methods of Thien and Lin [1] and Chan and Cheng [2], as an estimation of the MAE values for them.

**Lemma (Local distortion):** Let the base  $m \in \{2, 3, 4, 5, \dots\}$  be given and fixed. If we use Eq. (1) to embed a base- $m$  digit  $x$  in an integer value  $y$  to get  $y'$ , then

$$-0.5m < y' - y < 0.5m \text{ if } m \text{ is odd; } -0.5m < y' - y \leq 0.5m \text{ if } m \text{ is even. (7)}$$

**Mean absolute error (mae) theorem (the formula to predict MAE of Stego):** If the embedded data  $\{x\}$  is random, and the host has  $W \times H$  elements; then after embedding in each element of the host a message digit by Eq. (1), the expected value for the Mean Absolute Error (MAE), which is defined as

$$\begin{aligned} \text{MAE(Stego file, host file)} &= \sum_{i=1}^{W \times H} |y'_i - y_i| / (W \times H) \text{ is} \\ \text{MAE} &= 0.25 \times m - 0.25/m \quad \text{if } m \text{ is odd;} \\ \text{MAE} &= 0.25 \times m \quad \text{if } m \text{ is even.} \end{aligned} \quad (8)$$

(Proof of MAE Theorem)

If  $m$  is odd, then  $0.5m$  is not an integer; but  $0.5(m-1)$  is. The Lemma states that  $-0.5m < y' - y < 0.5m$ . From this and the case-by-case proof of the Lemma, we see that in a modulus- $m$  system, the possible values of  $y' - y$  are the  $m$  consecutive integers  $\{-0.5(m-1), \dots, -2, -1, 0, 1, 2, \dots, 0.5(m-1)\}$  that satisfying the  $-0.5m < y' - y < 0.5m$  requirement. The random distribution makes each integer here has similar occurrence rate ( $1/m$ ). After taking absolute values and doing average for these  $m$  values, we get

$$\begin{aligned} &[|-0.5(m-1)| + |-0.5(m-1)+1| + \dots + |-2| + |-1| + 0 + 1 + 2 + \dots + (0.5(m-1)-1) + 0.5(m-1)]/m \\ &= 2[1+2+ \dots + (0.5(m-1)-1) + 0.5(m-1)]/m = [1 + 0.5(m-1)] \\ &(0.5(m-1))/m = ((0.5m)^2 - 0.25)/m = 0.25m - 0.25/m. \end{aligned}$$

The case that  $m$  is even can be treated likewise. The  $m$  possible values of  $y' - y$  are  $\{-(0.5m - 1), \dots, -2, -1, 0, 1, 2, \dots, 0.5m - 1, 0.5m\}$ . After taking absolute values and then doing average,  $\text{MAE} = (0.5m + 0 + 2[1+2+3+\dots+(0.5m - 1)]) / m$

$$= (0.5m + 0 + (0.5m - 1 + 1)(0.5m - 1)) / m = 0.25m$$

-End of Proof for MAE Theorem-

**Proof of Lemma:** Let  $a = (y)_{\text{mod } m}$ , then the original host value  $y$  satisfies

$$y = nm + a \quad (9)$$

for some nonnegative integer  $n$ . Since both integers  $a$  and  $x$  are in the range  $\{0, 1, \dots, m-1\}$ , then, as stated earlier, the possible values of  $a-x$  are

$$a-x \in \{-m+1, -m+2, \dots, -2, -1, 0, 1, 2, \dots, m-1\}.$$

Therefore,  $-1 < (a-x)/m < 1$  holds. Hence,  $\text{round}((a-x)/m) \in \{-1, 0, 1\}$ .

Then,

$$y' = \text{round}\left(\frac{y-x}{m}\right) \times m + x = \text{round}\left(\frac{nm+a-x}{m}\right) \times m + x = (n + \text{round}\left(\frac{a-x}{m}\right)) \times m + x$$

is either

$$(n-1)m+x, \text{ or } nm+x, \text{ or } (n+1)m+x. \text{ Together with Eq. (1) and (9),}$$



we have

$$y' - y = (\text{round}\left(\frac{y-x}{m}\right) \times m + x) - (m + a) = -(a-x) + \text{round}\left(\frac{a-x}{m}\right) \times m \quad (10)$$

**Case 1**

If  $0.5m < (a-x) < m$ , then  $\text{round}((a-x)/m) = \text{round}(0.5\dots) = 1$ . Eq. (10) implies that  $y' - y = -(a-x) + m$ , which is a value in the range  $(-m + m, -0.5m + m)$ , i.e.  $0 < y' - y < 0.5m$ .

**Case 2**

If  $-0.5m < (a-x) < 0.5m$ , then  $\text{round}((a-x)/m) = 0$ . Eq. (10) implies that  $y' - y = -(a-x)$ , which is a value in the range  $-0.5m < y' - y < 0.5m$ .

**Case 3**

If  $-m < (a-x) < -0.5m$ , then  $\text{round}((a-x)/m) = -1$ . Eq. (10) implies that  $y' - y = -(a-x) - m$ , which is a value in the range  $(0.5m - m, m - m)$ , i.e.  $-0.5m < y' - y < 0$ .

**Case 4**

If  $a = x \pm 0.5m$ , which occurs only if  $m$  is an even number (because  $a$  and  $x$  are both integers), then  $y' - y = 0.5m$ , as proved below. There are two situations:

4a: if  $a = x + 0.5m$ , then (1) and (9) give us

$$y' = \text{round}\left(\frac{y-x}{m}\right) \times m + x = \text{round}\left(\frac{m+a-x}{m}\right) \times m + x = \text{round}\left(\frac{m+0.5m}{m}\right) \times m + x = \text{round}(n+0.5) \times m + x = (n+1)m + x = (m+a) + (x-a) + m = (y) - 0.5m + m = y + 0.5m$$

Therefore,  $y' - y = 0.5m$ . The other situation is

4b: if  $a = x - 0.5m$ , then Eq. (1) and (9) gives us

$$y' = \text{round}\left(\frac{y-x}{m}\right) \times m + x = (\text{round}\left(\frac{m+a-(a+0.5m)}{m}\right)) \times m + (a+0.5m) = (\text{round}(n-0.5)) \times m + (a+0.5m) = n \times m + (a+0.5m) = (m+a) + 0.5m = y + 0.5m$$

Therefore,  $y' - y = 0.5m$ , again.

-End of the proof for the Lemma-

**Results and Discussion**

First we check whether the MAE formula Eq. (8) is good or not. We use four cover images {Lena, Baboon, F16 (Jet), Peppers}, each is 512-by-512, as host images. For each host image, we do ten experiments: five of the ten experiments use random data as the secret files, and the other five experiments use some natural images as the secret files. Table 3 shows the average of the ten experiments for each host image and for each  $m$ . We can see that the prediction formulas of MAE and PSNR are very useful because they are simple (e.g. MAE=0.25m if  $m$  is even; MAE=0.25m-0.25/m if  $m$  is odd) and very close to the experimental values. The PSNR values predicted by Property 3 [1], i.e. Eq. (3-4) here, is also very close to the experimental PSNR values. Notably, MAE=0.25m (or 0.25m-0.25/m if  $m$  is odd) means that the absolute error at each pixel is small. For example, when  $m=4$ , i.e. when  $\text{bpp} = \log_4/\log_2 = 2$ , the MAE is MAE=0.25m=1; and hence, the absolute distortion  $|y'-y|$  at each pixel value  $y$  is 1 in average. Likewise, when  $m=8$ , i.e.  $\text{bpp} = \log_8/\log_2 = 3$ , the MAE is MAE=0.25×8=2; and hence, the absolute distortion  $|y'-y|$  at each pixel of gray value  $y$  is 2 in average. Note that the gray value  $y$  is in the range 0-255; so a distortion like 2 is not large.

Below is a comparison of Methods [1,2] with some other methods reported recently. Since both Methods [1] and [2] can be expressed

as Eq. (1-2) (except that Ref. [1] has a wider range of application because  $m$  is not necessarily in {2,4,8,16}), below we treat Methods [1] and [2] as a single method, i.e. the method using Eq. (1-2). Since this simple method emphasizes simplicity, high hiding capacity, and low distortion; we compare it with some other recently-published methods which also have high hiding capacity and low distortion.

Kanan and Nazeri [4] elegantly proposed a tuneable hiding method in spatial domain based on a genetic algorithm (GA). Their main idea is modeling the steganography problem as a searching-and-optimization problem. Then, in order to avoiding the exhausting searching of the optimized answer, a genetic algorithm is used in the searching of optimization. Their experimental results demonstrated that, in comparison with other popular steganography techniques, their algorithm not only achieves high embedding capacity but also enhances the PSNR of the stego images. According their Table 6, their PSNR values are about 45.63 dB when the hiding rate was 1.96 bits per pixel (bpp); whereas PSNR values of Thien and Lin [1] are about 46.37 dB at 2 bpp. Notably, larger bpp and larger PSNR are usually preferred, because larger larger bpp means higher capacity for hiding, and larger PSNR often means better stego image quality. Similarly, Kanan and Nazeri [4] has PSNR= 36.0 dB when bpp is 3.2; whereas Thien and Lin [1] has PSNR near 38.84 dB when bpp is 3.32. Finally, PSNR of Kanan and Nazeri [4] is near 35.01 dB when bpp is 3.95; whereas PSNR of Thien and Lin [1] is about 34.81 dB when bpp is 4 (or, about 35.42 dB when bpp is 3.91). Hence, the simpler and faster method (just using Eq. (1) and (2) for coding and decoding, resp.) of Thien and Lin [1] can compete with the Gene-Algorithm-searching optimization method introduced in study by Thien and Lin [4].

The data hiding method proposed by Tang et al. [5], is a gorgeous reversible method based on maximizing the difference values between neighbouring pixels. The conclusion of [5] stated that "...gains good PSNRs. compared to the scheme INP [7], the experimental results of proposed information hiding scheme improved the capacity. Therefore, this shows fully the better performance of the proposed scheme." Now, if we compare the simple method (our Eq. (1-2)) here with the high-capacity method introduced in Ref. [5] which uses multilayer embedding; then, according to Tables 1 and 2 of Tang et al. [5], the scheme in [5] can offer an average payload 1.79 bpp and average PSNR value 33.85 dB, whereas the simple method (Eq. (1-2)) here can offer average PSNR value 46.37 dB if 2 bpp (i.e.  $m=4$  in Eq. (1-2)) is used. Again, the PSNR-bpp performance of the simple method using Eq. (1-2) is competitive. But, of course, their method has the reversible ability to recover the host image, and this is a feature that we do not have.

Below we compare the 2003 simple method (Eq. (1-2)) with a sophisticated modulus-based method [6] introduced by Maleki et al., in 2014. There are two versions in [6]: adaptive and non-adaptive. According to their experiments for their non-adaptive version [6], their PSNR values are 51.12-51.18 dB for 1 bpp; 46.35-46.38 dB for 2 bpp; 40.65-40.75 dB for 3 bpp; 34.75-34.89 dB for 4 bpp. These values are very close to the {51.14 dB; 46.37 dB; 40.73 dB; 34.81 dB} values predicted by our Eq. (4) here, i.e. very close to the values predicted by Property 3 of [1], with  $m=2,4,8,16$ ; respectively (or equivalently, with  $\text{bpp}=1,2,3,4$ , respectively).

As for the adaptive version of Maleki et al. [6], their bpp can be non-integer; and PSNR values for Maleki et al. [6] are as in Table 1. We can see that the simple method here (Eq. (1-2)) is still very competitive.

Besides Maleki et al., [6], the method of Sadashiv and Rao [8] is

also an adaptive method because in [8] each smoother block hides lesser number of bits compared to edged block. Our PSNR-bpp performance also can compete with this adaptive method [8]. For example, when the cover image is the Couple image, their Table 3 shows that they obtain

{(44.70dB at 1.95bpp); (36.96 dB at 2.80bpp); (36.85db at 3.20bpp)};

whereas our simple method that uses Eq. (1) for hiding can generate

{(46.3dB at 2 bpp); (39.8dB at 3.17bpp); (38.1dB at 3.46bpp)}.

As a remark, Mean Absolute Errors (MAE) are not shown in most of the published papers (including the above papers) which hide data in gray value images, so we only compare the PSNR values in the above paragraphs. However, the readers are welcome to use our MAE table here to compare with the MAE of their new methods. Also, to give the readers some ideas of the MAE, we compare our MAE with the MAE obtained by the Raja et al., [9] or El-Emam [10]. Since El-Emam [10] study is for color images, the image Lena has 512\*512\*3 bytes, so the bpp should be explained as number of hidden bits per pixel byte. Therefore, just like dealing with gray-level Lena, we are still talking about how many hidden bits there are in one host byte. From this table, we can see that the method of Eq. (1) is also competitive in terms of MAE (smaller MAE is better, larger bpp is better).

Finally, as a conclusion, from the above comparison with the methods introduced in [4-10], we can see that the capacity-distortion performance of the simple method [1,2] is still very competitive nowadays. Therefore, we can say that the method is simple in coding/decoding (see Eq. (1-2)), with low-distortion; of high hiding capacity, and equipped with simple formulas to predict excellently both MAE and PSNR of the stego images in a very easy manner.

#### Acknowledgement

The author would like to thank the reviewers for the valuable suggestions. These suggestions are appreciatively acknowledged. This work is supported by the grant MOST103-2221-E-009-119-MY3.

#### References

1. Thien CC, Lin JC (2003) A simple and high-hiding capacity method for hiding digit-by-digit data in images based on modulus function. *Patt Recogn* 36: 2875-2881.
2. Chan CK, Cheng LM (2004) Hiding data in images by simple LSB substitution. *Patt Recogn* 37: 469-474.
3. Chan CK, Cheng LM (2001) Improved hiding data in images by optimal moderately-significant-bit replacement. *IEEE Elect Lett* 37: 1017-1018.
4. Kanan RH, Nazeri B (2014) A novel image steganography scheme with high embedding capacity and tunable visual image quality based on a genetic algorithm. *Expert Syst Appl* 41: 6123-6130.
5. Tang M, Hu J, Song W (2014) A high capacity image steganography using multi-layer embedding. *Optik - Int J Light Elec Optics* 125: 3972-3976.
6. Maleki N, Jalali M, Jahan MV (2014) Adaptive and non-adaptive data hiding methods for grayscale images based on modulus function. *Egy Inform J* 15: 115-127.
7. Lee C, Huang Y (2012) An efficient image interpolation increasing payload in reversible data hiding. *Expert Syst Appl* 39: 6712-6719.
8. Sadashiv S, Rao P (2015) A steganography method by using adaptive algorithm for gray scale images. *Int J Elect Electronics Comp Syst* 03: 09-13.
9. Raja KB, Vikas, Venugopal KR, Patnaik LM (2006) High capacity Lossless secure image steganography using wavelets. *Int Conf Advanced Computing and Communications* 230-235.

10. El-Emam N (2015) New data-hiding algorithm based on adaptive neural networks with modified particle swarm optimization. *Comp & Security* 55: 21-45.
11. Willmott CJ, Matsuura K (2005) Advantages of the mean absolute error (MAE) over the root mean square error (RMSE) in assessing average model performance. *Climate Research* 30: 79-82.
12. Chai T, Draxler RR (2014) Root mean square error (RMSE) or mean absolute error (MAE)? –Arguments against avoiding RMSE in the literature. *Geosci Model Dev* 07: 1247-1250.
13. George LE, Saleh HH, Hassan NF (2010) Data hiding in audio file by modulating amplitude. *Eng & Tech J* 28: 941-958.
14. Bhanushali S, Lad S, Haria V, Bhogale P (2015) Computer-aided breast tumor segmentation. *Int J Computer Applications* 115: 33-36.

#### Author Affiliation

Top

<sup>1</sup>Professor, Department of Computer and Information Science, National Chiao Tung University, Hsinchu, Taiwan 300, Taiwan

#### Submit your next manuscript and get advantages of SciTechnol submissions

- ❖ 50 Journals
- ❖ 21 Day rapid review process
- ❖ 1000 Editorial team
- ❖ 2 Million readers
- ❖ Publication immediately after acceptance
- ❖ Quality and quick editorial, review processing

Submit your next manuscript at • [www.scitechnol.com/submission](http://www.scitechnol.com/submission)