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Extending the Applicability of an Ulm-Newton-like Method under Generalized Conditions in Banach Space

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Abstract

The aim of this paper is to extend the applicability of an Ulm-Newton-like method for approximating a solution of a nonlinear equation in a Ba-nach space setting. The su cient local convergence conditions are weaker than in earlier works leading to a larger radius of convergence and more precise error estimates on the distances involved. Numerical examples are also provided in this study. AMS Subject Classi cation: 65H10, 65G99, 65J15,49M15.

Keywords

Ulm's method; Banach space; local / semi-local conver-gence

Introduction

In this study we are concerned with the problem of approximating a locally unique solution x of equation

$$F(x) = 0; \tag{1.1}$$

where, F is a Frechet{di erentiable operator de ned on a convex subset of a Banach space B1 with values in a Banach space B2.

A large number of problems in applied mathematics and also in engineering are solved by nding the solutions of certain equations. For example, dynamic systems are mathematically modeled by di erence or di erential equations, and their solutions usually represent the states of the systems. For the sake of sim-plicity, assume that a time{invariant system is driven by the equation x = R(x), for some suitable operator R, where x is the state. Then the equilibrium states are determined by solving equation (1.1). Similar equations are used in the case of discrete systems. The unknowns of engineering equations can be func-tions (di erence, di erential, and integral equations), vectors (systems of linear or nonlinear algebraic equations), or real or complex numbers (single algebraic equations with single unknowns). Except in special cases, the most commonly used solution methods are iterative{when starting from one or several initial approximations a sequence is constructed that converges to a solution of the equation. Iteration methods are also applied for solving optimization problems. In such cases, the iteration sequences converge to an optimal solution of the problem at hand. Since all of these methods have the same recursive structure, they can be introduced and discussed in a general framework [1-12].

Moser [13] proposed the following Ulm's-like method for generating a sequence fxng approximating x :

$$x_{n+1} = x_n - B_nF(x_n), \qquad B_{n+1} = 2B_n - B_nF'(x_n)B_n.$$

Method (1.2) is useful when the derivative F 0(xn) is not continuously invertible (as in the case of small divisors [1-8, 10, 11, 13-15]). Moser studied the semi-localpconvergence of method (1.2) and showed that the order of convergence is 1 + 2 if F 0(x?) 2 L(B2; B1p). However, the order of convergence is faster than the Secant method (i.e. 2). The quadratic convergence can be obtained if one uses Ulm's method [14,15]

$$x_{n+1} = x_n \quad B_nF(x_n)$$

$$B_{n+1} = 2B_n \quad B_nF^0(x_{n+1})B_n.$$

The semi-local convergence of method (1.3) has also been studied in [1-9]. As far as we know the local convergence analysis of methods (1.2) and (1.3) has not been given. In the present paper, we study the local convergence of the Ulm's-like method de ned for each n = 0; ; 2; 3; : : : by

$$x_{n+1} = x_n - B_nF(x_n,)\ B_{n+1} = 2B_n - B_nA_{n+1}B_n,$$

where A_n is an approximation of $F'(x_n)$. Notice that method (1.4) is inverse free, the computation of $F^0(x_n)$ is not required and the method produces suc-cessive approximations $\{B_n\} \approx F'(x^*)^{-1}$

In Section 2, we present the local convergence analysis of the method (1.4) and in Section 3, we present the numerical examples.

Local convergence analysis

The local convergence analysis of the method (1.4) is given in this section. Denote by $U(v, \zeta)$ the open and closed balls in $\boldsymbol{B}_1$, respectively, with center $v \in \boldsymbol{B}_1$ and of radius $\zeta>0$.

Let $w_0 : [0,+\infty] \to [0,+\infty]$ and $w : [0,+\infty] \to [0,+\infty]$ be continuous and nondecreasing functions satisfying $w_0(0)= w(0)=0$.

Let also $q \in [0,1]$ be a parameter. Define functions ϕ and ψ on the interval $[0,+\infty]$ by

$$\phi(t) = [q(\int_0^1 w(\theta t)d\theta + 1) + w_0(t)]t$$

and

$$\psi(t) = \phi(t) - 1$$

We have that $\psi(0) = -1$ and for sufficiently large $t_0 \geq t, \psi(t_0) > 0$. By the intermediate value theorem equation $\psi(t) = 0$ has solutions in the interval $(0, t_0)$. Denote by the smallest such solution. Then, for each $t \in [0, \rho]$ we have

$$0 \leq \psi(t) < 1. \tag{2.1}$$

We need to show an auxiliary perturbation result for method (1.4).

LEMMA 2.1 Let $F : \Omega \subseteq B_1 \to B_2$ be a continuously Frechet-differentiable operator. Suppose that there exist $x_* \in \Omega, \{M_n\} \in L(B_2, B_1), \{q_n\}, q \in R_0^+$, continuous and nondecreasing functions $w_0 : [0,+\infty) \to [0,+\infty)$ and w: $[0,+\infty) \to [0,+\infty)$ such that for each $x \in \Omega, n = 0,1,2,..$ and $\theta \in [0,1]$

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$F(x_*)=0, F'(x_*)^{-1}\in L(B_2,B_1),$

$\| F'(x_*)^{-1}F'(x_*+\theta(x-x_*))-F'(x_*))\| \le w(\theta\| x-x_*\|),$

$\| F'(x_*)^{-1}F'(x)-F'(x_*))\| \le w(\theta\| x-x_*\|),$

$\| F'(x_*)^{-1}(A_n-F'(x_n))\| \le q_n \| F'(x_*)^{-1}F(x))\|$

that for each $x, x_n \in \Omega_0 := \Omega \cap B(x_*,\rho),$

sup $q_n \le q,$

where $n\ge 0$

$x_n \in B(x_*,r_0),$

and $B(x_*,r_0)\subseteq \Omega,$

where $r_0\in(0,\rho).$

Then, the following items hold

$$\| F'(x_*)^{-1}F'(x_n)\| \le (\int_0^1 w(\theta\| x_n-x_*\|)d\theta+1)\| x_n-x_*)\|,$$

$$\| F'(x_*)^{-1}(A_n-F'(x_n)]\| \le q(\int_0^1 w(\theta\| x_n-x_*\|)d\theta+1)\| x_n-x_*\|,$$

$A_n\in L(B_2B_1)$

And

$$\| A_n^{-1}F'(x_*))\| \le \frac{1}{1-\phi(\| x_n-x_*\|)}$$

Proof we shall first show estimate (2.11) holds. Using (2.1), we have the identity

$$F'(x_n)=F(x_n)-F(x_*)=F(x_n)-F(x_*)-F'(x_*)(x_n-x_*)+F'(x_*)(x_n-x_*)=\int_0^1[F'(x_*+\theta(x_n-x_*))-F'(x_n)](x_n-x_*)d\theta \tag{2.14}$$

Then, by (2.4) and (2.14) we have that

$$\| F'(x_*)^{-1}F(x_n)\| \le \int_0^1 \| F'(x_*)^{-1}[F'(x_*+\theta(x_n-x_*))-F'(x_*)]\| d\theta \| x_n-x_*\| + \| x_n-x_*\|$$

$$\le (\int_0^1 w(\theta\| x_n-x_*\|)d\theta+1)\| x_n-x_*\|,$$

which shows (2.10). Moreover, by (2.5), (2.6) and (2.10) we obtain that

$$\| F'(x_*)^{-1}[A_n-F'(x_n)]\| \le q_n \| F'(x_*)^{-1}F(x_n)\| \le q(\int_0^1 w(\theta\| x_n-x_*\|)d\theta+1)\|x_n-x_*\|$$

which shows the estimate (2.11). Furthermore, using (2.3), (2.4), (2.10), (2.11) and the definition of r_0 we get that

$$\begin{aligned}&\| F'(x_*)^{-1}[A_n-F'(x_*)]\| \le \| F'(x_*)^{-1}[A_n-F'(x_n)\| + F'(x_*)^{-1}[F'(x_n)-F'(x_*)]\| \\ &\le \phi(\| x_n-x_*\|) \\ &\le \phi(| r_0)<1\end{aligned} \tag{2.15}$$

It follows from (2.15) and the Banach lemma on invertible operators [1, 4, 6, 11] that (2.12) and (2.13) hold.

REMARK 2.2 In earlier studies the Lipschitz condition [1-15]

$$\| F'(x_*)^{-1}[F'(x)-F'(y)]\| \le w_1(\| x-y\|) \text{ for each } x,y, \in\Omega \tag{2.16}$$

is used which is stronger than our conditions (2.3) and (2.4). Notice also that since $\Omega_0\subseteq\Omega,$

$$w(t)\le w_1(t) \tag{2.17}$$

and

$$w_0(t)\le w_1(t), \tag{2.18}$$

where functions w_1 is as function w but defined on Ω instead of Ω_0. The ratio $\frac{w_0}{w_1}$ can be arbitrarily large [1, 4, 6]. Moreover, if (2.16) is used instead of (2.3) and (2.4) in the proof of Lemma 2.1, then the conclusions hold provided that r0 is replaced by r_1 which is the smallest positive solution of equation

$$\psi_1(t)=0, \tag{2.19}$$

where $\phi_1(t)=\phi_1(t)-1$ and $\phi_1(t)=\left[q\left(\int_0^1 w_1(\theta t)d\theta+1\right)+w_1(t)\right]t$ it follows

from (2.10), (2.17), (2.18), (2.19) that

$r_1\le r_0$

Furthermore, strict inequality holds in (2.20), if (2.17) or (2.18) hold as strict

Inequalities. Finally, estimates (2.11) and (2.12) are tighter than the corre-

sponding ones (using (2.16)) given by

$$\left\|F'(x_*)^{-1}[A_n-F'(x_n)]\right\| \le q(\int_0^1 w_1(\theta\left\|x_n-x_*\right\|)d\theta+1)\left\|x_n-x_*\right\|.$$

Let λ be a parameter satisfying be a continuous and no decreasing function.

$\lambda\in[0,1]$

$[(0,p)\to[0,\infty),\beta[0,p)\to[0,\infty), f:[0,p)\to[0,\infty) and g:\beta$

Moreover, define functions

$$[(0,p)\to[0,\infty), by \alpha(t)=\frac{1}{1-\phi(t)}, \beta(t)=2q(1+$$

$$\int_0^1 w(\theta t)d\theta)t+2w_0(t), f(t)=)\alpha(t)\beta(t)-\lambda g(t)=$$

$$\lambda 2+(1+\lambda 2)\alpha(t)\int_0^1 w_2(1-\theta)t)d\theta-1$$

$$\alpha_k=\frac{}{-\ (\|x\ -x\ )}, \beta_k:=q(1+\int w\left(\theta\|x_k+1-x\ \|)d\theta\right)$$

$$\left\|x_{\ +1}-x_*\right\|+q(1+\int w(\theta\left\|x_k-x_*\right\|)d\theta)\left\|x_k-x_*\right\|+w_0(\left\|x_{k+1}-x_*\right\|)$$

$$+w_0\left\|x\ -x_*\right\|), d_0=\ _0$$

$$\gamma_k=\left\|I-B_K A_K\right\|^2+2\left\|I-B_K A_K\right\|\left\|A_{k+1}-A_k\right\|+\left\|B_k\right\|^2\left\|A_{k+1}-A_k\right\|^2$$

Parameters α,β by $\alpha=\alpha(r_0), \beta=\beta(r_0)$ ıd quadratic equatio $(1+\alpha\beta)t^2+2\alpha\beta(1+\alpha\beta)t+(\alpha\beta)^2-\lambda^2=0.$ Then, we have $f(0)=-\lambda<0 and f(t)\to\infty as t\to\overline{p}$

Denote by ρ_0 the smallest solution of equation *f*(*t*)=0 in (0,*p*) tthen, we have

that for each $t\in(0,\rho_0)$

$0<\alpha(t)\beta(t)<\lambda\cdot$

In view of the above inequality the preceding quadratic equation has a unique positive solution denoted by $\rho+$ and a negative solution. Define parameter γ by

$0\le\gamma\le\gamma_0=\min\{\rho+,\rho_0,r_0\}.$

Then, we have that

$(1+\alpha\beta)r^2+2\alpha\beta(1+\alpha\beta)\gamma+(\alpha\beta)^2<\lambda^2.$

Notice that we also have that $\alpha_k\le\alpha$ and $\beta_k\le\beta$.

Next, we present the local convergence of method (1.4).

THEOREM 2.3 Under the hypotheses of Lemma 2.1 and with r0 given in (2.9) for $\lambda \in [0,1)$ further suppose there exists function $w_2 : [0,r_0) \to [0,+\infty)$ continuous and no decreasing such that for each x $x \in B(x_*, r_o)\theta \in [0,1]$ and

$$\left\| A_n^{-1} \right\| \le \frac{1}{1-\phi(\|x_n - x_*\|)} < \phi_1(r_1).$$

$\|F'(x_*)^{-1}[F'(X_* + \theta(x-x_*)) - F'(x)]\| \le w_2((1-\theta)\|x-x_*\|)$ for each $x \in \Omega_0 = \Omega \cap B(x_*, r_0)$,

$\|I - B_0 A_0\| \le d_0 < \lambda^2$ and

$B(X_*, \gamma) \subseteq \Omega$

where γ is given in (2.22). Then, sequence $\{x_n\}$ generated by the method (1.4) for $x_0 \in B(x_*, \ ) - \{x_*\}$ is well defined , remains in $B(x_*, \gamma)$ and converges to x_*

Proof. We have by hypothesis (2.25) that $\|I - B_0 A_0\| \le d_0 < \lambda^2$ so

$\|I - B_K A_K\| \le \gamma_k < \lambda^2$

is true for k = 0: Suppose that (2.27) is true for all integers smaller or equal to k: Using Lemma 2.1, we have the estimate

$$\begin{aligned}
\|B_K\| &= \|B_K A_K A_K^{-1}\| \le \|B_K A_K\| \|A_K^{-1} A_K^{-1}\| \\
&\le (1 + \|I - B_K A_K\|) \|A_K^{-1}\| \\
&\le (1+\gamma_k) \frac{1}{1-\phi(\|x_k - x_*\|)} \le (1+\gamma_k)\alpha_k
\end{aligned}$$

In view of method (1.4) for n = k; we can write in turn that

$$\begin{aligned}
&\|F'(x_*)^{-1}(A_{k+1}) - A_K)\| \\
&\le \|F'(x_*)^{-1}(A_{k+1}) - F'(x_{K+1}))\| \\
&+ \|F'(x_*)^{-1}(F'(x_{K+1}) - F'(x_K))\| + F'(x_*)^{-1}(A_k - F'(x_*))\| \\
&\le \|F'(x_*)^{-1}(A_{k+1}) - F'(x_{K+1}))\| + F'(x_*)^{-1}(A_k - F'(x_K))\| \\
&+ \|F'(x_*)^{-1}(F'(x_{K+1}) - F'(x_*)\| + \|F'(x_*)^{-1}(F'(x_K) - F'(x_*))\| \\
&\le \|F'(x_*)^{-1}(A_{k+1}) - F'(x_{K+1})\| + \|F'(x_*)^{-1}(A_k - F'(x_K))\| + \|x_{K+1} - x_*\| \\
&\le q(1 + \int_0^1 w(\theta\|x_{k+1} - x_*\|)d\theta)\|x_{k+1} - x_*\| \\
&+ q(1 + \int_0^1 w(\theta\|x_k - x_*\|)d\theta)\|x_{k+1} - x_*\| \\
&+ w_0(\|x_k - x_*\|) + w_0(\|x_k - x_*\|) \\
&\le \beta_K \le \beta
\end{aligned}$$

By the definition of method (1.4), we have the estimate

$I\text{-}B_{k+1}A_{k+1} = I\text{-}(2B_k\text{-}B_kA_{k+1}B_k)A_{k+1} = (1\text{-}B_kA_{k+1})^2$

Then, by (2.32), (2.29) for $n = k$; we get in turn that

$$\begin{aligned}
&\|I-B_{k+1}A_{k+1}\| \le (\|I-B_kA_k\| + \|B_k\|\|A_{k+1}-A_k\|)^2 \\
&\le (\|I-B_kA_k\|^2 + 2\|1-B_kA_k\|\|B_k\|\|A_{k+1}-A_k\| + \|B_k\|^2\|A_{k+1}-A_k\|^2 \\
&\gamma_k^2 + 2\gamma k(1+\gamma k)\|A_k^{-1}\|\|A_{k+1} - A_k\| \\
&+ (1+\gamma k)^2 \|A_k^{-1}\|^2 \|A_{k+1} - A_k\|^2 \\
&\le \gamma_k^2 + 2\gamma k(1+\gamma k)\alpha\beta + (1+\gamma k)^2 \alpha^2\beta^2 \\
&= (1+\alpha\beta)^2\gamma_k^2 + 2\alpha\beta(1+\alpha\beta)\gamma k + \alpha_k^2\beta_k^2 \\
&\le 1+\alpha\beta)^2\gamma^2 + 2\alpha\beta(1+\alpha\beta)\gamma + \alpha^2\beta^2 < \lambda^2
\end{aligned}$$

which shows (2.27) for $n = k+1$: Then, using the induction hypotheses, (2.24), and the definition of γ

$$\begin{aligned}
&\|x_{k+1} - x^*\| \le (\lambda^2 + (1+\lambda^2)\alpha\| x_k - x_*\| \\
&\times \int_0^1 \omega_2((1-\theta)\|x_k - x_*\| d\theta \|x_k - x_*\| \\
&< g(\gamma)\|x_k - x_*\| \le g(\rho_+)\|x_k - x_*\| c \|x_k - x_*\|,
\end{aligned}$$

where $c = g[0,1]$, so $\lim_{k\to\infty} x_k = x_*$ and $x_{k+1} \in B(x_*,\rho)$

REMARK 2.4 (a) As noted in Remark 2.2 conditions (2.4) and (2.5) can be replaced by (2.24).

$$\|F'(x_*)^{-1}[F'(x_* + \theta(x - x_*)) - F'(x)]\| \le \omega_3((1-\theta)\|x - x_*\|) \quad (2.36)$$

for each $x \in \Omega$ and $\theta \in [0,1]$, where function ω_3 is as ω_1:

We have that $\omega_1(t) \le \omega_3(t)$. Then, in view of Remark 2.2 and (2.24) the radii of convergence as well as the error bounds are improved under the new approach, since old approaches use only (2.36) with the exception of our approach in [2, 5].

The results obtained here can be used for operators F satisfying autonomous differential equations [1, 4, 6, 11] of the form

$F'(x) = P(F(x))$

Where $P : \square \to \square$ is a continuous operator. Then, since $F'(x_*) = P(F(x_*)) = P(0)$, we can apply the results without actually knowing x_*. For example, let $F(x) = e^x - 1$. Then, we can choose $P(x) = x + 1$

(c) The local results obtained here can be used for projection methods such as the Arnoldi's method, the generalized minimum residual method (*GM-RES*), the generalized conjugate method (*GCR*) for combined Newton/finite projection methods and in connection to the mesh independence principle can be used to develop the cheapest and most efficient mesh refinement strategies [1,4,6].

(d) Let L_0, L, L_1, L_2, L_3 be positive constants. Researchers, choose $\omega_0(t) = L_0 t$, $\omega(t) = Lt$, $\omega_1(t) = L_1 t$, $\omega_2(t) = L_2 t$, and $\omega_3(t) = L_3 t$, Moreover, if we choose $\Omega_0 = \Omega$ and $L = L_1$ then, our results reduce to the ones given by where the second order of convergence was shown with the Lipschitz conditions given in non-affine invariant form. In Example 3.1, we shall show that the radii are extended and the upper bounds on $\|x_n - x_*\|$ are tighter if we use ω_0, ω, ω_2 instead of using ω_0 and ω we used in [5] or only ω_3 as used in [2,7-15].

Numerical examples

Example 3.1 let $X = \square^3, D = \bar{U}(0,1), x^* = (0,0,0)^T$ Define function F on D for $\omega = (x,y,z)^T$ by

$$F(\omega) = (e^x - 1, \frac{e-1}{2}y^2 + y, z)^T$$

Then, the Frechet-derivative is defined by

$$F'(\upsilon) = \begin{bmatrix} e^x & 0 & 0 \\ 0 & (e-1)y+1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Notice that using the Lipschitz conditions, we get $\omega_0(t) = L_0 t$, $(t) =$

Lt, $\omega_1(t)= L_1 t$, $\omega_2(t)= L_2 t$, and $\omega_3(t)= L_3 t$, where $L_0= L =$ e-1, $L_1= L_3=$e and $L_2= \frac{1}{e^{L_0}}$ Moreover, choose $A_n = \frac{1}{2}F'(x_n)$ to obtain $q_n=q=\frac{1}{2}$

The parameters are $\rho = 0.5758, r_1 = 0.4739, \bar{\rho} = 0.5499, \bar{r}_1 = 0.4739$

where the bar answers corresponding to the case when only ω_3 is used in the derivation of the radii.

Example 3.2 Let

$X = Y = \square^{m-1}$ for natural integer

$n \geq 2$ X and Y are equipped with the max-norm x = max1≤i≤n-1x_i. The corresponding matrix norm is

$$A = \max_{1\leq i\leq m-1}\sum_{j=1}^{j=m-1}|aij|$$

For A= (aij)1≤i≤m-1. On the interval [0; 1], we consider the following two point boundary value problem

$$\begin{cases} \upsilon'' + \upsilon^2 = 0 \\ \upsilon(0) = \upsilon(1) = 0 \end{cases} \quad (3.1)$$

[6, 8, 9, 11]. To discretize the above equation, we divide the interval [0; 1] into m equal parts with length of each part: h=1/m and coordinate of each point: x_i=I h with i=0,1,2,…,m. A second-order finite difference discretization of equation (3.1) results in the following set of nonlinear equations

$$F(\mathrm{v}) := \begin{cases} \upsilon_{i-1} + h^2\upsilon_i^2 - 2\upsilon_i + \upsilon_{i+1} \\ for i = 1,2,...,(\mathrm{m}-1)\mathrm{and from}(3.1)\upsilon_o - \upsilon_m = 0 \end{cases} \quad (3.2)$$

Where V=$[\upsilon_1, \upsilon_2, ..., \upsilon_{(m-1)}]^{\mathrm{T}}$ For the above system-of-nonlinear-equations, we provide the Frechet derivative

$$F'(\mathrm{v}) = \begin{bmatrix} \frac{2v_1}{m^2}-2 & 1 & 0 & 0\cdots0 & 0 \\ 1 & \frac{2v_2}{m^2}-2 & 1 & 0\cdots0 & 0 \\ 0 & 1 & \frac{2v_3}{m^2}-2 & 1\cdots0 & 0 \\ \vdots & \vdots & \vdots & \vdots\cdots\vdots & \vdots \\ 0 & 0 & 0 & 0\cdots1 & \frac{2v_{(m-1)}}{m^2}-2 \end{bmatrix}.$$

We see that for

$$A_n = \frac{9}{10}F'(x_n)$$

$\omega_0(t)$=$L_0 t$, $\omega(t)$=Lt, $\omega_1(t)$=$L_1 t$, $\omega_2(t)$=$L_2 t$, and $\omega_3(t)$=$L_3 t$, where L_0=L=L_1=L_2=3, q=$\frac{1}{10}$ and $\| F'(x_*)^{-1} \| = \frac{1}{2}$ The parameters are

$$\rho = 0.5478, r_1 = 0.5478, \bar{\rho} = 0.4762, \bar{r}_1 = 0.4762$$

where the bar answers corresponding to the case when only ω_3 is used in the derivation of the radii.

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